# 11 Physical distinctions for decision processes

In this chapter and the next we continue our analysis using specific numerical models to understand the new distinctions that arise in the *decision process theory*. Some of these distinctions are physical, which we deal with in this chapter. Some of the distinctions are more social, which we deal with in the next chapter.

The physical distinctions can be attributes that are directly reflected in the differential geometry. An underlying assumption of *decision process theory* is that it applies to processes which have a degree of continuity similar to other physical processes so that tools of topology and geometry are applicable. There are several distinct mechanisms that mutually interact in this theory so the theory is expected to provide insights that are new and distinct from the several diverse starting assumptions. These rely on distinctions that are common to a wide variety of phenomena that we characterize as physical. We start with a particular attribute of such models which is their structural stability.

## 11.1 *Morphogenesis and structural stability*

We assume processes are connected both in space and in time: strategic choices made by one group impact choices another group makes. We have pointed out that the resultant equations, which are hyperbolic partial differential equations, exhibit a mathematical stability that small changes in the initial conditions result in small changes in the solutions (Wald, 1984). We also expect that such solutions are characterized by a ***structural stability*** or ***locality*** property that the structural character of a solution does not change dramatically as long as we stay in the near vicinity of that solution.

The expectations of *structural stability* are local not global attributes of nature or of *decision process theory*. In nature we have phenomena on a lake or an ocean that on a calm day, exhibit no wave motion. The components of the lake are connected. As soon as there is a small breeze, we see small ripples on the water. This is an example of the expected structural stability. As the breeze picks up however, the ripples turn into waves that at some point exhibit *breakers*, which are new structures. This morphogenesis follows from rather general arguments and can be expected whenever we have a theory based on topology and geometry of the type we have been considering (Thom, 1975). We expect such morphogenesis to occur in nature as well as in any sufficiently realistic theory and will follow from the differential equations. We expect the phenomena of ripples to result from the fluid equations of the theory of fluid dynamics. Indeed, such equations are used to predict the structural anomalies of hurricanes and tornadoes in meteorology (Friedman, 1989). In like fashion we expect to see structural changes in *decision process theory*. In this section we inquire into such ***morphogenesis*** and investigate their possible mechanisms.

The possibility of structural instability in decision processes was raised in the classic work that investigated the limits of global growth (Meadows, Meadows, Randers, & Behrens, 1972). However it is not often the focal point of economic theories. For example game theory is concerned with the equilibrium state and so implicitly concerned only with the small ripples that might occur nearby. It is not ordinarily concerned with the possible *breakers* that occur when the wind picks up. We are concerned with such phenomena for a couple of reasons. First, such phenomena could provide confirmation or condemnation for the theory. Second, such phenomena may be of practical interest to either identify or potentially avoid. Like hurricanes, such phenomena may be destructive so knowing when they are imminent is useful information.

To make these arguments more quantitative, in our numerical solutions in earlier chapters, we constructed examples of structural change. So for example in section 5.10, we demonstrated that if the *stakes* in the prisoner's game were sufficiently high, we obtained a dramatic change in the behavior of the *decision process value*, Figure 5-36. In section 6.8.4, for the Robinson Crusoe model, we found a similar result if we introduced a large strain, Figure 6-21 and Figure 6-22. Again with *quasi-stationary solutions* in the section 9.5 for the attack-defense model, we saw more examples when the *harmonic* behavior associated with acceleration effects was large.

*Decision process theory* admits solutions that are perturbations of *game theory* equilibrium behavior. We have seen this in (Thomas G. H., 2006) and in our numerical examples in this book. In particular, for the *player fixed frame models*, with a little algebra using the definitions and Eq. (9.12), as well as exercise 17 from section 8.12, we wrote the frame equations Eq. (3.32) as Eq. (8.78). From these equations, we glean that for behavior at the *still point*, we have zero *active geometry acceleration*: the *still point* behavior is then set by $\Omega_{\upsilon\upsilon'}$:

$$\begin{aligned}
&Q_\upsilon = \frac{q_\upsilon + 2\omega_{\upsilon\alpha}e^\alpha + \omega_{\upsilon\alpha\beta}e^\alpha e^\beta}{\left(1+e_\alpha e^\alpha\right)^2} = 0 \qquad (11.1)\\
&\chi \equiv \frac{1}{1+e_\alpha e^\alpha}\\
&\Omega_{\upsilon\upsilon'} = \omega_{\upsilon\upsilon'} + \tfrac{1}{2}E_{ko}f^k{}_{\upsilon\upsilon'} = \left(\omega_{\upsilon\upsilon'} - e^\alpha\omega_{\alpha\upsilon\upsilon'}\right)\chi = -\tfrac{1}{2}\mathrm{e}^{-q}\chi f_{\upsilon\upsilon'}\\
&\Delta_o E^a{}_o = 0\\
&\Delta_o E^a{}_\upsilon = -\Omega_{\upsilon\upsilon'}E^{a\upsilon'}
\end{aligned}$$

This represents the simple structure analogous to ripples on a lake. Specifically, when there are no tidal forces (*free fall*) and no charges, the flows are constant and the spatial components of the frame rotate with components $-\tfrac{1}{2}\mathrm{e}^{-q}\chi f_{\upsilon\upsilon'}$, the *characteristic payoff*. The *characteristic payoff* acts identical to the Coriolis force and reflects a rotating *central co-moving frame* (*Cf.* the form of the metric, Exercise 45, Chapter 4). Moreover we see that the *absolute force* is also zero. In the special case that the *characteristic payoff* is zero, the *composite payoff* is just the weighted average of the player payoffs and corresponds to the figures in (Thomas G. H., 2006).

The assumption of no tidal forces can be imposed at any point by choice of gauge: it reflects *characteristic payoff* acceleration or *Coriolis* forces. Ordinarily this will not be true at all points as viewed in the same reference frame. For example it might not be true along an initial streamline, even though along that streamline we can take the metric to be Minkowski and make simple model choices for the orientation potentials, so that they are small or correspond to a static gravity field. Indeed, these choices describe the basic assumptions or models used in (Thomas G. H., 2006). They provide a baseline structure on which we can look for new behaviors (*Cf.* Figure 9-26).

For example, we ignored the fact that the inactive scalar fields and the payoff fields may vary along the streamline (*Cf.* Exercise 52, Chapter 4). More generally, we look at the *free fall harmonic* behavior as only providing a guide and any non-zero *harmonic* behavior as a steady-state wave pattern. This corresponds to the lake being flat with ripples: the interesting question is what happens when the amplitude of the ripples becomes large. Some ripples are a direct consequence of the *free fall* behavior and others are a consequence of additional *acceleration effects*. A complete description is to use the full set of equations with the appropriate boundary conditions.

A behavior when the ripples become strong is shown in the section 9.5, exercise 11, Figure 9-24 from an earlier chapter. The behaviors cease to be characterized by the same structure only modified by larger amplitudes, but are ***morphogenic***. The study of *morphogenic* behaviors brings in other attributes of the theory that themselves may represent simple concepts but are now interacting together in a complex way. We see in a dramatic way that mechanisms in *decision process theory* are ***connected***. Such mechanisms bring in new distinctions. What are these *connections* that change structure and do we have evidence to support the existence of these *connections*?

In subsequent sections, we inquire into the *decision process theory's* mechanisms that change structure. In particular, we identify a set of new standalone features predicted by *decision process theory* that we believe are worthy of further investigation.

## 11.2 *Causality*

In section 7.3 we argued that decision processes are ***causal*** not stochastic, despite the fact that decisions involve uncertainty. We argued strongly against the Bayesian approach (Bayes, 1764) and for an approach looking at possible decision choices as a frequency distribution that evolves in time in a causal manner. We illustrate these arguments with a common problem in software development: delivering a complex project on time. We fabricate a scenario below, though from experience, a scenario that we believe does in fact happen.

The delivery of a complex project, one that involves perhaps 500 developers, reflects a large number of decision processes. It is attractive to management to think of such processes as occurring stochastically and to believe that the delivery of a complex project will follow the same pattern of previous projects of the same scale and complexity. We imagine that a company X has gone through 4 or 5 delivery cycles, each taking 2 years and each having been delivered on time. The projection of the next delivery is thus expected by management to take the same time, 2 years. But a variation occurs; after development has started, the customer asks for new functionality, essentially increasing the size of the project. Let's say both the customer and company X agree that the increase size is 20% and that the company agrees to hire 100 more people and the customer is willing to pay for the increased cost. Management predicts that the original schedule will thus be maintained.

The people are hired but to the dismay of the customer and management, the project ends up costing significantly more than the 100 people and does not come in on time, even though the additional staff were hired with sufficient time, based on past data, to get the job done. Both sides looked at the problem from the same worldview, that of a complex stochastic process. Neither side anticipated failure.

How would we look at this process and would the expected results be predicted to be different? A causal approach (ours or other similar causal approaches) would look for key dynamic mechanisms that govern the delivery of the software product. We would agree that hiring the additional people is a necessary ingredient. However we would find on analysis and by interviewing members of the technical staff that not all employees are equally productive. The previous releases were staffed by essentially the same people with only a small turnover. The skill level of that staff was constant: new hires that were needed as replacements were always added to the project only after adequate training. To satisfy the new customer request however, staff were added after the overall project was well underway. The additional 100 people were new employees that were not yet trained and so had a much lower skill level. Though many of the employees and lower level managers complained that they would not be able to deliver based on the addition of untrained staff, their voices were overruled since the management model had no way to use the skill level to modify their schedule prediction. They interpreted the employee complaints as normal grousing about being asked to go for a stretch goal.

The consequence of the lower skill level was in fact not felt immediately. The lower skill level introduced a higher level of defects into the product, but these were not detected early on. When the product was moved along to the testing phase, the higher number of defects was detected (per unit of code). The defect insertion rate could have been predicted based on existing data on new hires from past releases. The company of course was now in the testing phase and had no choice but to hire even more additional staff to find and fix the defects. They were also not able to roll off existing staff as planned to other releases. They hired an additional unplanned 50 people, though in fact that was enough only to fix the first round of defects. They spent 6 months additional because of this testing. Of course the additional people hired were again unfamiliar with the product and many of their fixes also broke. The failure rate of their fixes was about the same as the other new hires. The consequence of this problem was the fix-break time cycle.

Under normal circumstances a release was expected to pass all but 5% of its tests by the time it entered the final test cycle. The test cycle of 6 months was part of the expected 2 year delivery. That final cycle would reduce the number of defects by another 95%, an acceptable escaped defect rate. The reduction of defects was in fact a direct measure of the skill level and training of the employees. The new circumstance dropped that defect reduction percentage dramatically. They saw only a 50% reduction in defect rate.

For a 50% reduction, as they went into the final cycle, 50% of the tests would not pass. If they did a second cycle, they would have 25% that would not pass; a third cycle and 12% and a fourth cycle would be 6%. To get to where they should have been in past releases they would need a fifth cycle, which is four extra cycles. Each extra cycle cost them 6 months and 50 people. This extended the project by two years and added an unplanned 50 extra people on the staff, as well as keeping on the project staff that had been expected to shift to other releases. The total project cost should have been 500 people for 2 years: 1000 staff years. The budget office showed that the overall project cost was 2000 staff years.

Now the example above is fictitious but illustrates how even a very simple exercise of adding a single causal mechanism (staff expertise) has a profound impact on the outcome. Time loops can be very expensive in time and money. They are also relatively invisible to the participants of the decision process. The decision about what features to deliver to the customer followed standard form, albeit late in time. The decision about when the project was complete did not change. From a stochastic view the late project and the normal projects are similar. Indeed, it is easy to make the assumption that all behaviors are stochastic. The alternative approach of modeling the behavior as a causal processes seems harder as it requires identifying the dynamic mechanisms.

Despite being complicated however, it has been observed that such dynamic mechanisms are often well understood by organizations (*Cf.* section 7.8). In this example, employees and managers knew that a lower skill level would lead to a higher defect rate. In a stochastic model, there was no place to put that information. The real difficulty is that the global consequence of such mechanisms is not easily deduced without a causal framework. Within a causal framework (e.g. the attack-defense model section 9.1 modified by exercise 1 at the end of this chapter) we obtain global structures that we think would be hard to anticipate or visualize, Figure 11-1 and Figure 11-2.

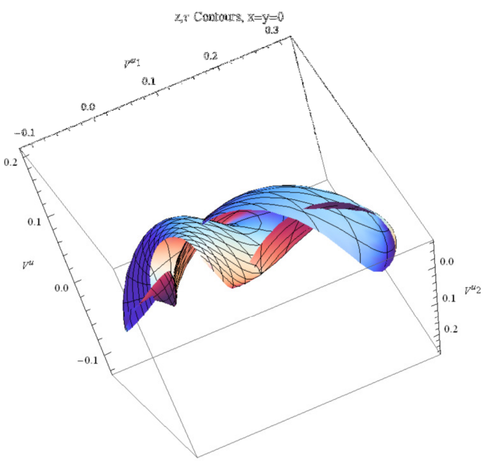


**Figure 11-1: Attack-defense active flow contours** $V^a(0,0,z,\tau)$

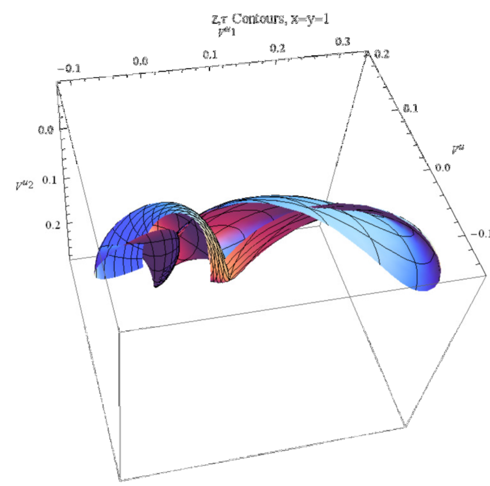


**Figure 11-2: Attack-defense active flow contours** $V^a(1,1,z,\tau)$

A focus on dynamic mechanisms leads us to look not just at decisions as events but to look at the stresses and strains that impact decision making and the timing for these events. We look at effects that may slow down or speed up decision processes. In the above case, the skill level of new hires slowed the decision process. An outcome of our inquiry into decision processes using *decision process theory* is the identification of many such mechanisms.

### 11.3 *Aggression and limits to greed and accommodation*

Decision processes are engaged in by players that may engage more or less strongly as a function of the dynamics. If one player engages more strongly than all the others as indicated by frequency of making decisions we say that player is acting ***aggressively***. The extreme opposite, zero *engagement*, is that the player is acting ***passively***. To act at all requires an ***act of will*** and so must require some *aggressive* or *accommodating* behavior.

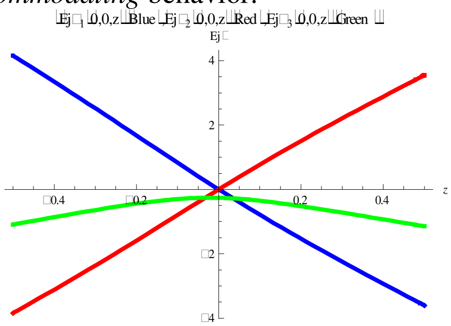

**Figure 11-3: Attack-defense model *engagement* (charge) values $E_{jo}(0,0,z)$; "attack" (red), "defense" (blue) and code of conduct (green)**

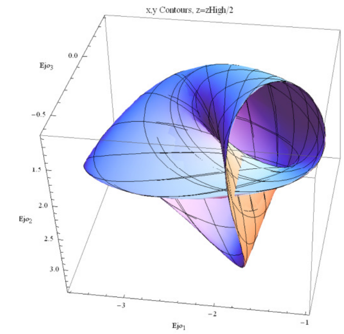

**Figure 11-4: Attack-defense model *engagement* (charge) contours $E_{jo}(x,y,\frac{1}{4})$ where "defense" is aggressor**

In the attack-defense model with a small acceleration frequency (section 9.1 modified by section 11.9 exercise 1), we see that such *aggressive* behavior is tied to a ***greedy*** (negative) engagement or charge, Figure 11-3. The other player is tied to an ***accommodating*** (positive) charge. We demonstrated similar behavior for the prisoner's dilemma, section 5.8.3. The new feature with three active strategies is that we see a limit both to the amount of *greed* and the amount of *accommodation*, Figure 11-4.The same effect is seen in the co-moving frame, Figure 11-5 and Figure 11-6.

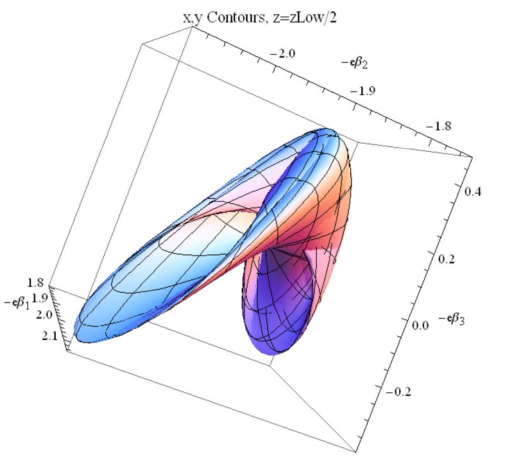


**Figure 11-5: Attack-defense co-moving frame charge contours $e^{\alpha}(x,y,-\frac{1}{4})$ where "attack" is aggressor**

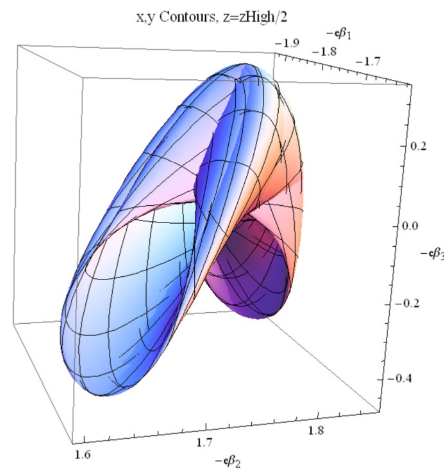


**Figure 11-6: Attack-defense co-moving frame charge contours $e^{\alpha}(x,y,\frac{1}{4})$ where "defense" is aggressor**

In many ways the result reflects common sense. Not everyone can be the *aggressor* and not everyone can be the *accommodator*. The world does not naturally separate into groups of people who will always be accommodating to those that are greedy. Because we live in a social network, there are limits to how greedy or how accommodating we will be as a society. That is reflected in *decision process theory*. In part, the origin of the limitation is the possibility of *harmonic solutions* with a growth limited to be linear in proper time. More generally we think such limitations must be present in a theory that completely represents all aspects of the process.

For our specific models here, the limitation can be seen in the elasticity of the decision process system. In an elastic system, stressing the components by displacing them (generating strain) produces a bounce of the system as a response; the system changes other parts in a continuous manner in such a way as to balance out the applied force. We have applied an input force as a consequence of setting the boundary conditions and now observe the consequences to the system in these figures. The elasticity provides a limitation to greedy behavior or accommodating behavior by either party.

The limits to *greed* or *accommodation* can perhaps not be proved but can be taken as an operating ***neutrality principle***. There are limits to the amount of ***polarization***. In *decision process theory*, strong *engagement* is either *greed* or *accommodation* depending on the numerical sign of the engagement. Another aspect of the limits of *greed* (*accommodation*) is the existence of a neutral point of zero *engagement* that attracts other behaviors. The physical analogy is that in general, matter that occurs in nature is electrically neutral. Though it is possible for matter to be strongly polarized, that is not its "natural" state. On the other hand, even though matter is neutral, we can't ignore the effects of polarization. We see these effects as radiation, magnetization, electrical permittivity *etc*. We don't confuse limits on polarization and ground state neutrality as a statement of the dynamic condition. The existence of a neutral point is the source of the elastic dynamic behaviors.

We have demonstrated the virtues of ***locked behavior*** (section 9.2), which supports this *neutrality principle*. We have solutions that have zero *acceleration* and *player engagements* at one point. At this point the solution concentrates the energy density supporting the notion that the preferred place to be is at this neutral point. So for example if each player has a positive (negative) player bias, the generalization of game value, then when that player is more *aggressive*, then the *engagement* is negative (positive).

For *locked behavior* we see the *engagement* grows with the size of *aggression.* We remember however that we are looking at *stationary* flow so we are not arguing that the *aggression* grows dynamically. We are also not saying that aggression tends to zero to the neutral point. What we do say is that there is a correlation of *aggression* with *engagement.* A consequence of this correlation is that the energy density is clumped, at least for these *locked* solutions (*Cf*. Figure 11-14). As part of the *stationary* flow, we have a range of values of aggression and engagement.

### 11.4 *Network connectivity*

Decision processes occur in a societal context, so that decisions are connected to each other not only across time but across strategic distance. The limit of greed (section 11.3) is an example. We have normally envisioned the societal context as that of multiple groups doing similar things at about the same time. In this way the network is similar to a fluid in physics. However, there is a significant difference between human behaviors and those of inanimate objects: humans remember. Thus the same individual may see similar situations a variety of times and respond not just on present circumstances but on those events that have happened in the past. In this way the individual sees a network based on his or her ***experience*** *as* well as on a network based on the observations of what others are doing currently. We allow both meanings of network in what follows. Both contribute to the societal or network connectivity.

The significance of ***network connectivity*** (Barabási, 2003) is that such connectivity plays a key role in behavior. Moreover it is deterministic not random. The internet provides a prime example of the importance of *network connectivity*. The network distinctions give rise to ***morphogenic changes*** such as phase transitions. There are many examples of *network connectivity*. These network distinctions are new as they don't arise in *game theory*. Network connectivity gives rise to *morphogenic changes*. Diverse local behaviors can combine to form new and unexpected global structures such as Figure 11-5 and Figure 11-6. We have computed other examples of this in section 5.10 where high stake and low velocity behaviors generated trapped behaviors, Figure 5-36.

As a practical example, when Japanese practices dominated manufacturing because of adoption of Deming's management method (Walton, 1986), manufacturing in the United States was not unaffected. Indeed those practices returned to the United States and caused an effective revolution in quality for US made products. Adopting techniques currently being used by someone else is a *network connection*. We are not waiting for another decision to be made by ourselves; we are mimicking what someone else is doing because they are in a similar space at a neighboring point in time. These are ***gradient effects*** over the network, which propagate causally. An analogy is the pressure gradients in weather. The flow of air reflects the causal nature of the weather. We effectively describe the weather however by considering not the flow of air but rather the areas of high pressure and low pressure. The pressure gradients (as seen over distance) from high pressure to low pressure generate the forces, which help us understand the causal motion of the air. These gradients reflect the elastic network connectivity of the atmosphere.

In a similar fashion we look for ***pressure gradients*** in decision processes. They measure the network connectivity forces. We look for evidence of ***network compression*** of strategic areas. Areas that compress should be under higher pressure than those that don't. The ability of a strategic area to resist compression, its ***bounce***, is a measure of the ***elasticity*** of the decision process in that area. Decision processes reflect a complex scenario of network interactions. The measure of *network connectivity* will be based on the change between players executing a particular strategy and other players executing a neighboring strategy.

Again as a practical example, when a company comes out with a revolutionary and successful product, we might well believe that it is the result of causal activities that happen solely within that company. That product might reflect new technologies or new uses of existing technologies developed by that company. The product is revolutionary in that it is not a mirror of any existing product. Once released however, we often see many companies simultaneously imitating that product. This is reflective of the *network connectivity* rather than a causal train of events in each of these companies based on their own innovation and design.

Our use of the distinction of *pressure gradient* fits in with our general framework of units, section 1.6. We distinguish strategies as the ***effort*** that goes into making a choice and ***utility*** or value of that choice. *Utility* and *effort* are not equivalent in *decision process theory*, analogous to *stress* and *strain*, respectively, in physics; each generates the other through dynamic mechanisms. We have identified three mechanisms that change *utilities* (sections 7.5, 7.6 and 7.7): pair-wise ***competition***, pair-wise ***cooperation*** and ***narrative*** (context, story or ***opportunity***). Each are associated with forces that change behaviors and create high density and low density behaviors: ***capital accumulation*** and ***capital loss***. *Network connectivity* is a mechanism for change that is a consequence of these capital fluctuations. Accumulated capital must be spent and lost capital must be replenished.

*Capital,* including *experience*, therefore represents stress that is either constructive or destructive. For example, constraining the public's liberties generates stress, which we normally consider destructive. A company's creative ability generates wealth is a stress, which we normally consider constructive. Stress causes things to happen. *Capital* must be created or spent. Deprivation of freedoms can stimulate revolutions. Failures can stimulate growth. Creativity creates new opportunities. If there are stresses we should ask what efforts will now be undertaken and what strains will now occur. The loop must be closed. Stresses cause strain just as strains generate stresses. This leads us to the following hypothesis about a structure that might be stable.

***Successful decision structures*** are built on three key attributes: effort, utility and capital. They correspond to the definition of success as the ***three W's***: work, wisdom and wealth (Wikipedia; Henry Wriston, 2012). We characterize *successful decision structures* as those that are sustained and *stationary*, as those in which each player's efforts realize their vision (value or utility) and those in which the network flows result in *locked behaviors* with *capital accumulation* corresponding to the collective visions.

## 11.5 *Still point and free fall*

We believe that decision processes behave in a way analogous to electrical engineering systems in which there is characteristic impedance and a load impedance on which one imposes a forced behavior from a source: the ***characteristic payoff*** reflects the network attributes of the system; the *harmonic steady-state waves* (next section) reflect the source; and the load is determined by the initial conditions.

To fully understand the system we need to look at the static or characteristic behaviors as well as the dynamic behaviors.

Thus, just because we look for dynamic mechanisms, we don't ignore static behaviors and the consequences that are implied. An important static behavior to incorporate we believe is based on the *game theory* analysis of decisions. It provides a framework for the space in which we operate, the space of pure strategies extended to include mixed strategies as frequencies (sections 7.4 and 7.5). The theory will have solutions in which behavior along a path will have a ***still point*** (section 8.7), which will appear as one that has no *active geometry acceleration* forces: these are the sum of the *absolute force*, *competitive* or *game theory force* and the *cooperative force*. *Game theory* identifies such behavior as Nash equilibrium. Though there are differences in how each theory computes the composite behavior in terms of individual player behavior, both theories provide essentially the same distinction. We can use the insights learned from game theory while remaining consistent with the ideas of *causality*, without resorting to Bayesian probability as a predictor for future behavior, which is also consistent with at least one view of game theory (Von Neumann & Morgenstern, 1944).

However there is an important distinction between our approach and *game theory*. *Game theory* assumes greater freedom in how different specifications of strategies for a given decision problem lead to identical results. *Decision process theory* says that the theory can be expressed in a *covariant* way that is independent of the strategy specifications, but the numerical values may in fact depend on that specification and non-identical results. For any given specification one must take into account *acceleration* effects. The acceleration effects may manifest themselves as flows that change in time or as *Coriolis* effects in which the flows appear fixed but fictitious forces appear that reflect the fact that the description is in a frame that is rotating.

Thus we argue that at the *still point* there is no acceleration and away from that point there may be acceleration. We take this as the covariant specification. The numerical results and the mechanisms depend on the frame of reference. In some frames (such as the *central holonomic frame*, section 3.6), the active strategy flows are independent of time (indeed, in the *central holonomic frame* for the *fixed frame model*, the space components are zero and the time component is unity) while by assumption, the *covariant* rate of change of the flow with time is not constant.

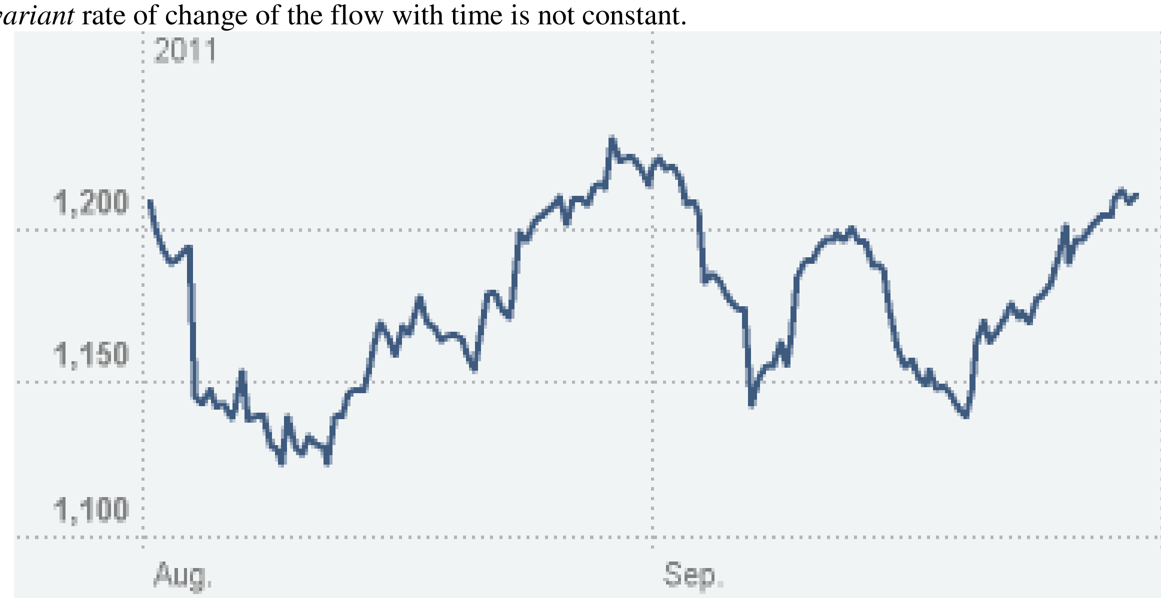


**Figure 11-7: New York Stock Exchange (New York Times, 2011)**

This is consistent with the common perception that such effects appear to be absent. We selected differential geometry as the basis for our theory because it addresses this issue. *Decision process theory* is built on the principle that at each point, the strategy space is locally flat: locally, one can find a coordinate system in which we appear to be at rest and there appears to be no acceleration. We distinguish this case by saying that in that specific coordinate system, we are in ***free fall*** at that point. However, this is only true to first order in the coordinates. The acceleration effects we are discussing are *frame rotation* effects and are of second order. The space is in fact curved, not flat, just as the earth appears locally flat but is a globe; the effects are second order. In decisions, we see a connectivity that is not only local but global so that we need differential geometry to stitch together the local pictures into a consistent global picture. We need to add the concept of *orientation flux fields.* As a consequence, our *central holonomic frame*, which provides a global view, in general is not locally flat, exercise 51 from chapter 4; in general, we are not in *free fall* in that frame, though we could arrange the initial conditions so that that would be true at that one point.

There are two types of behaviors that characterize a *still point*: those that are in *free fall* and those that are not. The latter can be resolved into ***harmonic steady-state wave*** behaviors (***forced behaviors***), with arbitrary frequencies, as a way to decompose all possible non-free fall behaviors. We say more about the latter in the next section. As a dynamic mechanism, *free fall* provides a *harmonic* structure with specific rotational frequencies set by the ***characteristic payoff***. Examples of this behavior have not been well studied. A challenge is to verify that the *harmonic* frequencies match the appropriate *free fall* values. As a start, consider a typical process reflected in the stock market behavior, Figure 11-7.

We are quite used to seeing fluctuations. If they are interpreted as random or stochastic, we are denying the possibility that the behavior might be causal. If we argue that the behavior is causal, then an alternate interpretation is that we are seeing multiple *harmonics*. The stock market might be overly complicated to start with, but the fact that it appears to have *harmonics* is encouraging. The market responds as if it was an elastic-object that has been struck. Specific events do appear to cause the market to oscillate and reverberate long after the event. A detailed study is needed to determine whether we can identify the *free fall harmonics* and separate them from the general *steady-state wave harmonics* we expect. The important new distinction is that we look not just at equilibrium behaviors but that we look at the *harmonics* that result after some impact event, such as in our model calculations for the attack-defense model Figure 9-27 and Figure 9-28.

Our model is quite different in several respects from the stock market. We have modeled a single type of market, whereas the stock market reflects many markets and many products. We imagine that the stock market reflects processes that go beyond our simple attack-defense example. In the real world, a market is not isolated but carries on in a larger environment. One way to model that environment is to ignore its effects as reflecting longer time scale phenomena. That is, we replace the real strategic variables by ones in which the environment effects are absent. For the stock market we would look at specific stocks and over limited periods of time. Only then might we see the free fall behaviors we are interested in. We would proceed somewhat like scientists investigating weather patterns by ignoring the spin of the earth (*Coriolis* forces) as a first approximation. At some point however we would return to the problem and add back such effects. That is our approach in considering *free fall harmonics* as separate from the more general *steady-state wave harmonics* considered in the next section.

The *game theory* distinctions, though not new, are worth repeating as they are not exactly the same as the *decision process theory* distinctions. In *game theory*, decisions are characterized by pure strategy choices or mixtures of pure strategy choices reflecting some frequency distribution. In contrast, the *still point* and *free fall* behavior is determined by the way in which one sets the initial conditions. Those conditions could be to require certain equilibrium values or could be to require certain initial player values (chapter 8). What we can say is that for *free fall*, there will be a mixed strategy choice that sets the rate at which each strategy changes. There will also be a *characteristic payoff* matrix that will determine the *harmonic* frequencies. Additional dynamic mechanisms from the theory determine the detailed relationship of these *free fall* values and initial boundary condition values for the players.

We distinguish between the player's *worldview* of payoffs that they use to make their decisions and the composite *worldview* that reflects the decisions that actually have been made. In *free fall* when there are no forces other than this composite *worldview*, the strategy choices reflect rotations. Indeed we argue more generally that the concept of payoffs, individually or composite, reflect some type of rotation. We name the *free fall* rotation ***decision vorticity***. It represents the rotation due to the *characteristic payoff*, and in a fixed frame is the weighted average of the *player payoffs*. Such a *characteristic payoff* can be represented by its eigenfrequencies based on the general properties of rotations.

Qualitatively, this generalizes Nash equilibrium. If players act on the basis of their individual payoffs, then Nash argues that there will be an equilibrium choice that is in some sense best in that no player can do better because of what other players would do in response. The Nash equilibrium is thus an optimal strategy for the *characteristic payoff*. Conversely, if we know the Nash equilibrium we imagine a variety of rotations that leave this direction unchanged, any one of which is a possible *characteristic payoff*. We say that the collective behavior will always be a rotation, so there will always be a *characteristic payoff* that measures that rotation. For any rotation, there will always be directions that are left fixed. Decision choices along such a direction correspond to Nash equilibriums.

### 11.6 *Harmonic steady-state wave*

In section 7.7, we suggested that one source of utility consists of *opportunity*: namely choices that we are not doing. Thus in addition to *free fall*, we may have sources of energy that have nothing to do with the market that we are in. Technically we look at this possibility as introducing an external jolt to the system.

Consider the following. At each point in space, the flows (and more generally the frames) can be represented as a superposition of *harmonics*. We can decompose the strategic flow at the *still point* in terms of *free fall harmonics* in which the frequencies are fixed in terms of the payoffs, and more general ***harmonic steady-state waves*** in which the frequencies are arbitrary. For example, to the attack-defense model we add a *harmonic steady-state wave* to the model reflected in Figure 9-27 and Figure 9-28. We obtain Figure 11-8 and Figure 11-9, where the added frequency is about a third that of *free fall*, exercise 1. Recall that we define the *still point* behavior as that which occurs when there is no ***active acceleration*** $Q_v = 0$, Eq. (8.78): collectively the *absolute forces*, *competitive forces* and *cooperative forces* cancel.

A typical system behavior is a stable behavior until an external event jolts it. The jolt can be represented by a characteristic superposition of *harmonic steady-state waves*. We provided an example of this in our software project example (section 11.2). The jolt in this case was the addition of new software requirements. Another example of a jolt might be the meltdown of the market due to toxic securities. Although it may be convenient to know the *free fall* behavior of the system, realistic systems are usually more complicated because they have been jolted. We must include the ability to jolt the system or set up steady-state waves in the system to reflect realistic behaviors. *Harmonic steady-state waves* provide the dynamic mechanism for studying such behavior.

As the software project example illustrates, a jolt to the system may not generate the result expected by the players at the time. We must include other dynamic mechanisms to correctly compute the outcome. In our theory, we in fact do take into account these other dynamic mechanisms. We assign a particular *harmonic* behavior along an initial streamline and use the theory to compute the behaviors along all other streamlines. In this way we consider the *network connectivity* effects; in the case of the software project, such other effects were the learning curve of new hires. This corresponds to a streamline that was not associated with *free fall* behavior.

The advantage of looking at active geometry tidal effects as *harmonic steady-state waves* is that we can categorize effects depending on their time scale. If we consider effects that have a very long time scale compared to the considered decision process (the *characteristic payoff*), then we have a *harmonic* frequency that is small compared to the *characteristic payoff* [26]. Such low frequency waves might be things to ignore or might be significant hints of things to come. On the ocean, such low frequency rollers can indicate oncoming severe storms. We would need to know the amplitude of the effect as well as the frequency. Similarly, low frequency effects in the market might be long lead time effects; if they reflected boom-bust cycles they could represent stormy financial times.

[26] Actually, we compare to one of the typical *free fall* eigenfrequencies. For the case of three active strategies, there would be a single imaginary frequency, Eq. (8.80).

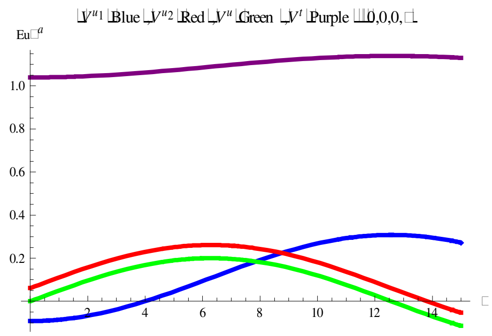

**Figure 11-8: Attack-defense input *harmonic* flows $V^a(0,0,0,\tau)$; attack (red), defense (blue), code of conduct (green) and time (purple) with $\omega = 1/4$**

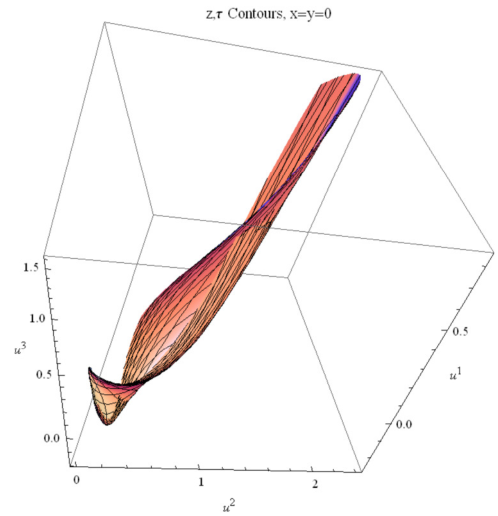


**Figure 11-9: Attack-defense coordinate contours $x^a(0,0,z,\tau)$ with $\omega = 1/4$**

Low frequency phenomena might be something we ignore. Do we have good evidence that that represents a good decision policy? Global warming is an example of something that has a long cycle time. Those that argue that global warming is significant still agree the effects are a long way off. They also agree the effects are currently small but growing. The issues that are in dispute are whether the amplitude will continue to grow and whether that growth is caused by humans. Whether significant growth is caused by humans or not, an observed large amplitude indicates changes in policy that will be needed to avoid serious consequences to the global economy; so one criterion for low frequency phenomena to be important is that their amplitude is large. In comparing different phenomena we can compare the product of the amplitude and the frequency. As the frequency goes down the amplitude must go up to make an equivalent effect.

The global warming example relies in part on an effect that continues to grow. There are effects however that are entirely cyclical and long term. An example is cyclical or seasonal market demands. At the end of the year, sales go up dramatically because of holiday buying. This skews the monthly market figures for members of the retail industry. It obviously changes their business strategies because the demand (amplitude) during the holiday is large. Such cyclical changes are important even though they don't generate exponential growth.

Short term fluctuations would correspond to high frequency *harmonic steady-state waves*. In the market place, these also might be seasonal effects or fads. It is clear that a robust dynamic picture requires inclusion of such effects. Such effects might dominate or mask other behaviors. The effects are typically not small. If such effects become sufficiently large, they may generate large *morphogenic changes*. We found evidence for such effects for the prisoner's dilemma (*Cf.* Figure 5-48). We can find similar effects for the attack-defense model, Figure 11-10 and Figure 11-11. To make the effects more visible, we increase the size of the strategy space by using the model parameters exercise 1 with player payoffs and related compression parameters reduced by $1/10$. We then see *attenuated frame waves*, which also generate ***gravity waves*** in the *normal form coordinate basis*.

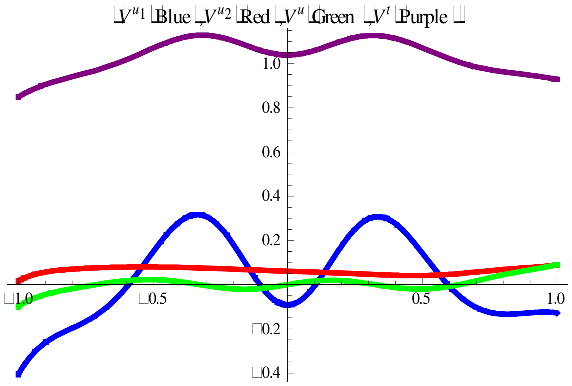

**Figure 11-10: Attack-defense flows at $\overline{\tau}=0$ versus $\overline{z}$ in the *central holonomic frame* with $\varpi=10$ and player payoffs reduced by $\frac{1}{10}$**

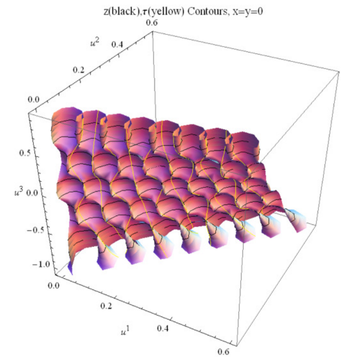

**Figure 11-11: Attack-defense coordinate contours $x^a\left(0,0,\overline{z},\overline{\tau}\right)$ in the *central holonomic frame* with $\varpi=10$**

The operative measure for short term fluctuations can again be taken to be the product of the frequency and the amplitude. Now however the frequency is large so even a small amplitude fluctuation can make an effect. For example consider the impact of advertising. If ads are run, often they can change attitudes and market, even when short. It is the frequency of the ads that makes an impact. This fact is observable during presidential elections. The ads need not address the issues so in that sense their frequency is much larger than what we have called the *free fall* frequency or *characteristic payoff*. It will be a consequence of other dynamic mechanisms to determine how these effects propagate. Advertising in fact changes the ***narrative***; it provides *orientation flux field* effects according to the *dynamic law of opportunity* (section 7.7).

In general, we expect to see a variety of effects: *free fall*, short term fluctuations and long term fluctuations. Returning to the stock market behavior example (Figure 11-7), we might imagine that the short term fluctuations reflect response to daily news events and information about individual stocks. The long term fluctuations might reflect trends in foreign markets. On top of this the general market behavior would reflect growth or decline based on the success of the market: the *free fall* behavior.

We suggest that the addition of forced oscillations does not change the definition of *still point*, only the frame of reference. We can view a *harmonic steady-state waves* change of reference as the addition of fictional forces to *free fall* behavior; the. For our attack-defense example the forced oscillations might represent cyclical behaviors outside of the context of the time frame of the attack-defense scenario. Nevertheless, such forces may not be needed if the strategies are expressed in some other frame of reference, just as *Coriolis* forces disappear in a frame of reference that looks at the earth's rotation from outside.

In our theory of an elastic system, forced oscillations drive the system and reflect a behavior with modified oscillations that characterize the underlying system behaviors. This has been exploited and is the basis for the analysis in Systems Dynamics (Meadows, Meadows, Randers, & Behrens, 1972). We note a more familiar example of bowing the strings of a violin. The sound that results is more than the oscillations of the strings. The sound reflects the character of the violin itself, so that one gets a sound from a Stradivarius that is more extraordinary than from an average violin. Thus starting from an input set of *harmonic* flows Figure 11-8, we obtain the resonant response Figure 11-9. We see that the response is much richer than the input.

We also look for empirical periodic behaviors. We know that there are numerous cycles in business. There are bust and boom cycles that have historical time frames measured in decades (Wikipedia, 2012) down to corporate project release cycles measured in weeks or months. In considering *free fall* behaviors, we should consider such behaviors as key characteristics of the decision process. Certainly one aspect of these different cycles is their length. The length is inversely proportional to the frequency and the frequency is proportional to the payoff. Assuming the same proportionality constant, high payoffs would correlate with short cycle time cycle lengths. Low payoffs would correlate with long time cycle lengths. We would be forced to consider their relative cycle lengths a dynamic feature of decision behaviors and correlate this with their payoffs along with the appropriate proportionality constants.

## 11.7 *Acceleration*

The outcome of the decision process consists of the actual choices made. The rates at which these choices are made represent the decision flow and are something that we directly measure. For example, we would look at survey questions as providing the current state of each player and what effort they would apply at that point in time. The survey is not predictive in the probability sense but provides information about the initial state; *decision process theory* then governs the evolution of that state. In this way we generalize the static flow considerations of *Game theory* to *stationary* flow and dynamic flow. In either case, the new distinction will be ***acceleration***; we expect the flow of decisions to change in direction, in magnitude or both. These changes provide a direct means to measure the new distinctions in *decision process theory*.

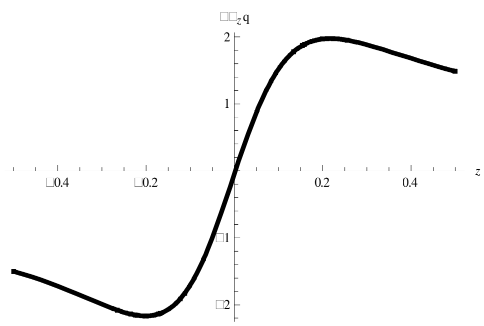


**Figure 11-12: Attack-defense locked behavior for acceleration gradient** $\nabla_z q(0,0,z)$

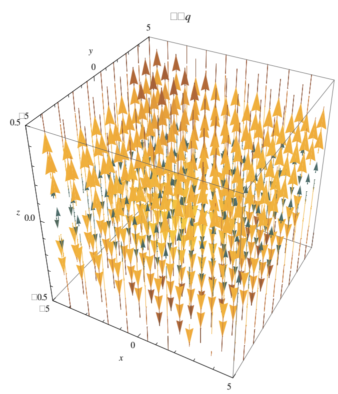

**Figure 11-13: Attack-defense locked behavior for acceleration gradient** $\nabla q(x,y,z)$

For example, we expect *stationary* flow to exhibit *harmonic steady-state wave* motion. The acceleration consists of decisions that are constantly changing their direction, even if their values are constant along their streamlines. We expect more complex *harmonic steady-state waves* in which the decision values also change. Acceleration is a new and important distinction.

We have freedom in choosing a frame of reference in which to observe the *acceleration*. Certain frames will make it easier to observe some features as opposed to others. We choose a ***co-moving frame***, one in which the flows are at rest, but where the acceleration is not necessarily zero. The *harmonic* motion is captured by the *characteristic payoff* and is not visible in the acceleration. However, other acceleration effects due to the interplay of forces in decision processes (*e.g.* the *locked* behavior, section 9.2) are more visible, Figure 11-12 and Figure 11-13. As decisions are made, we expect that choices in some strategic areas will be denser than in other areas. This generates pressure Figure 11-14 and compression Figure 11-15 that would push future choices away from those areas. The behavior of those choices would appear to *accelerate* from the high pressure areas to the low pressure areas. Such effects generate changes in momentum, a term often used in sports and in the stock market. We are familiar with team behavior improving suddenly as if they pick up outside support. We see their fortunes *accelerate*.

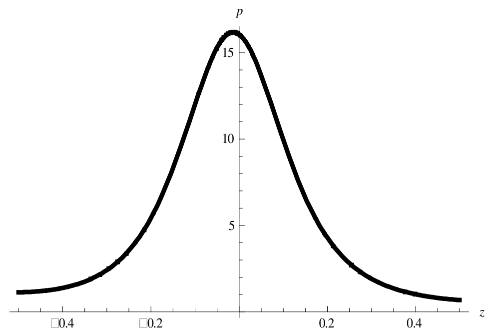


**Figure 11-14: Attack-defense locked behavior for pressure at** $p(0,0,z)$

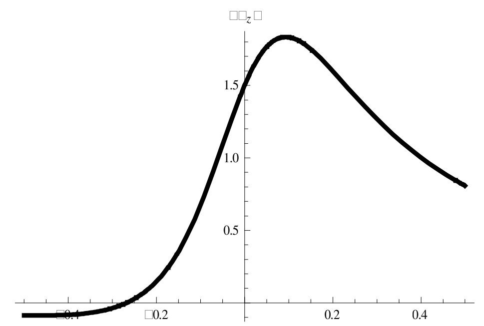


**Figure 11-15: Attack-defense locked behavior for compression gradient at** $\nabla_z \Theta(0,0,z)$

The opposite of *acceleration* is ***inertia***. Being slow to change ones decision strategies reflects large inertial effects. In fact there might be confusion over behavior that has settled into equilibrium versus behavior that is not changing because of inertia. We think of companies that are not making a profit but are set in their ways. There may in fact be winning strategies that they could adopt if it were not for the *inertia* and lethargy of the organization.

## 11.8 *Outcomes*

In this chapter we identify new distinctions generated by *decision process theory*:

1. ***Morphogenic***: Structures that we identify that are local and small may change into more complex structures as the fields increase in strength and interact with each other.
2. ***Causality***: Decision processes are causal not stochastic.
3. ***Aggression and limits to greed***: Decision processes are engaged in by players that may engage more or less strongly as a function of the dynamics. If one player engages more strongly than all the others as indicated by frequency of making decisions we say that player is acting *aggressively*. In model calculations, we see that *aggressive* behavior is tied to a ***greedy*** value (negative) for the engagement or charge. The other player (in a two person game) exhibits an ***accommodating*** (positive) value. We find that there are limits to both greedy and accommodating behaviors.
4. ***Network connectivity:*** Decision processes occur in a societal context, so that decisions are connected to each other not only across time but across strategic distance. This gives rise to a variety of distinctions associated with gradient effects: *pressure*, *network compression*, *bounce* and *elasticity*. In this concept we find support for the three W's: work, wisdom and wealth. We use these as the basis for ***successful decision structures***.
5. ***Still point*** and ***Free fall:*** *Decision process theory* will have solutions in which behavior along a path will have a *still point* (section 8.7) at which the *active geometry acceleration* is zero. This generalizes the notion of Nash equilibrium from *game theory* and allows us to use *game theory* as a baseline for composite decision behavior. A special case of still point behavior is one in which the strategies appear to be in a flat space; the acceleration is determined entirely by the rate of change of the strategy flow. This *free fall* behavior gives rise to *harmonic* behaviors with frequencies set by the *composite payoff*. Such frequencies are properties of the specific decision process.
6. ***Harmonic steady-state wave***: *Harmonic steady-state waves* can be used to drive the system and help illuminate the *free fall harmonics*. A general solution can have an arbitrary superposition of *harmonic steady-state waves*. The relative weights of the superposition are fixed by the initial conditions. A system can be studied by driving it with forced *steady-state wave harmonics*, which will illuminate the underlying *free fall harmonics*.

7. ***Acceleration***: We expect to see *acceleration* in decision processes; we expect the flow of decisions to change in direction or in magnitude. These changes may provide the most direct means to measure the new distinctions in *decision process theory*.

The student should be able to identify these distinctions in real world decision processes and qualitatively see how such distinctions would apply in the theory. The attainment of the outcomes of this chapter is facilitated by doing the exercises in the following section.

## 11.9 *Exercises*

1. Modify the attack-defense model (section 9.1 and section 9.5, exercise 7) to one in which we add to the *free fall harmonic,* an *acceleration behavior harmonic* with frequency $\omega = 1/4$. The payoffs remain the same as section 9.1:

$$\begin{aligned} G_{1(blue)} &= 1/10 \begin{pmatrix} 4 & 1 \\ 3 & 4 \end{pmatrix} \\ G_{2(red)} &= 1/10 \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix} \end{aligned} \tag{11.2}$$

2. For the attack-defense model used in this chapter, we think of the co-moving frame transverse active variable $z$ as a measure of aggression. As we dial up the value of $z$ we expect that the flow $V^u$ will increase, Figure 11-16. We accomplish this by fixing the initial value of $\partial_z V^u$. What should we expect for the gradients of the other flows $\{V^{u_1} \quad V^{u_2} \quad V^t\}$? Does it seem reasonable that if Blue were acting aggressively, he might start defending his more expensive target? Correspondingly might Red start attacking the less expensive target? What would reasonable values be for the gradients? See Figure 11-17.

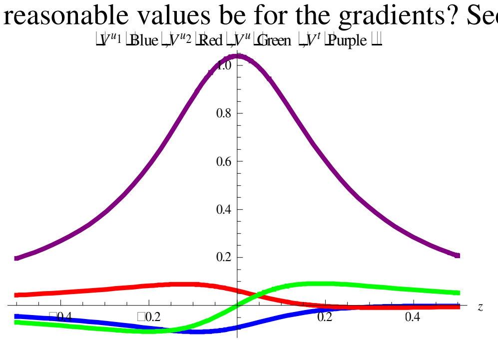


**Figure 11-16: Spatial behavior of the flow with initial aggression gradient** $\partial_z V^u = 1$

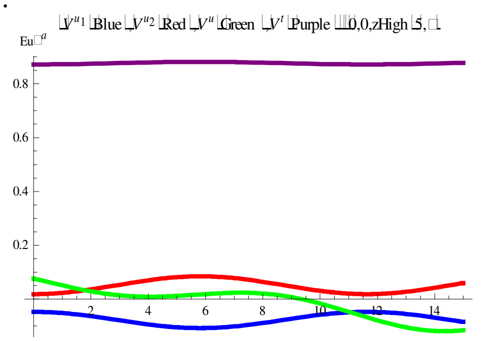


**Figure 11-17: Aggression gradient flows at** $z = 1/10$

3. Using exercise 2 we have computed the contour flows. Can you see how the aggression assumption plays out? Start with the proper time, Figure 11-18, where you should be able to see the effect of the initial gradient contribution that "widens" the shape of the figure. In Figure 11-19, you can see two different time *harmonics*; one is going up and down according to the flow along the aggression strategy and the other reflecting the *free fall* behavior.

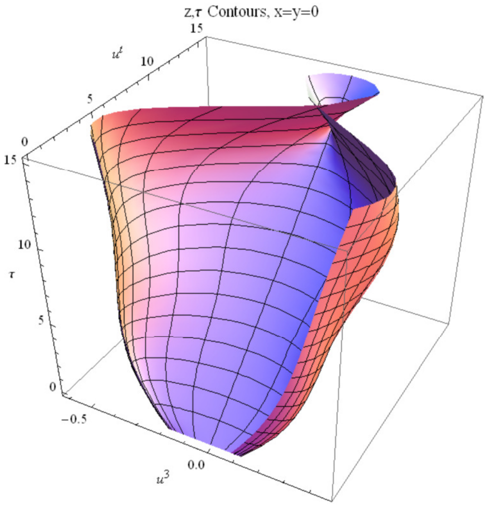


**Figure 11-18: Proper time for model with aggression**

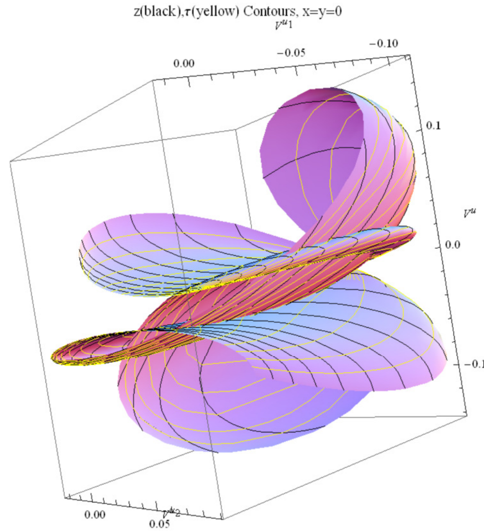


**Figure 11-19: Contour plot for flows with aggression assumption**

4. In exercise 1, we provided a *harmonic steady-state wave* with frequency $\omega = 1/4$, which is roughly a third the frequency of the *free fall* for a version of the attack-defense model. Consider the results below for a *harmonic steady-state wave* with frequency $\omega = 2$, which is roughly three times the frequency of the *free fall* version. Explain why the initial coordinate preferences for red and blue have reversed, Figure 11-20. Explain the structure for the coordinate contours, Figure 11-21. Why has blue now started to defend the more expensive target and red now attacking the less expensive one? Is this a prediction or an initial condition?

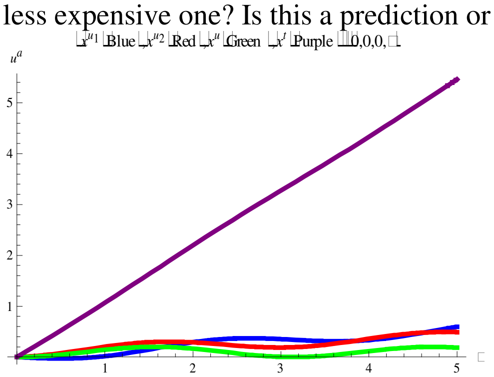

**Figure 11-20: Attack-defense initial coordinate *free fall* plus *harmonic steady-state wave* with $\omega = 2$**

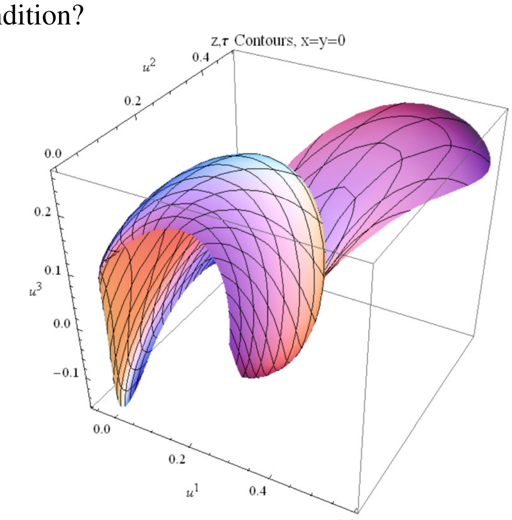


**Figure 11-21: Coordinate contours for attack-defense *free fall* with *harmonic steady-state wave* $\omega = 2$**

5. We consider the same model as in exercise 1 and provide the corresponding initial flows (Figure 11-22) and flow contours (Figure 11-23). Since the *free fall* flows are constants, we see only the *harmonic steady-state wave* in the first figure. Do we see evidence of the *free fall* flow in the second figure? Consider the proper time Figure 11-24. Can you characterize where the surfaces meet as morphogenic change?

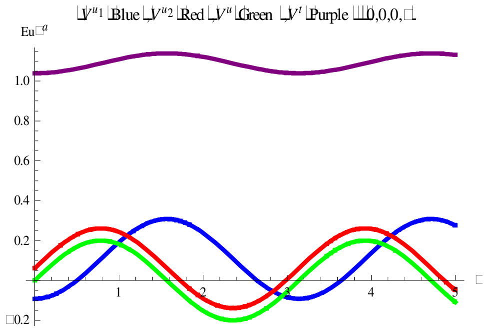


**Figure 11-22: Attack-defense initial flows for *free fall* plus *harmonic steady-state wave* with $\omega = 2$**

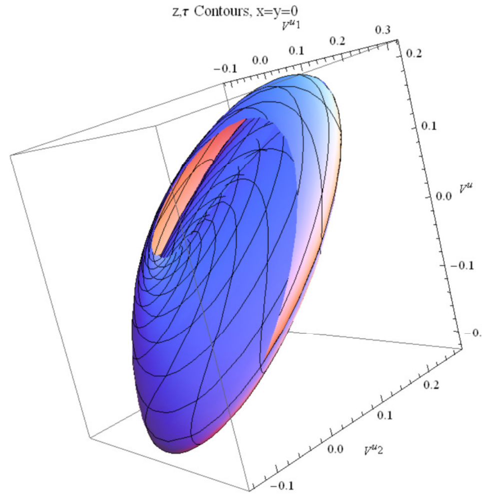


**Figure 11-23: Flow contours for attack-defense *free fall* with *harmonic steady-state wave* $\omega = 2$**

6. We consider the same model as in exercise 1 and provide the flow contours for $x = y = 1$, Figure 11-25. Can you characterize the shape as morphogenic change?

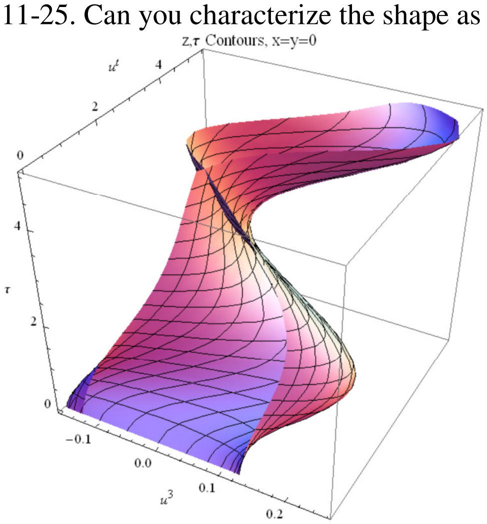


**Figure 11-24: Attack-defense proper time for *free fall* plus *harmonic steady-state wave* with $\omega = 2$**

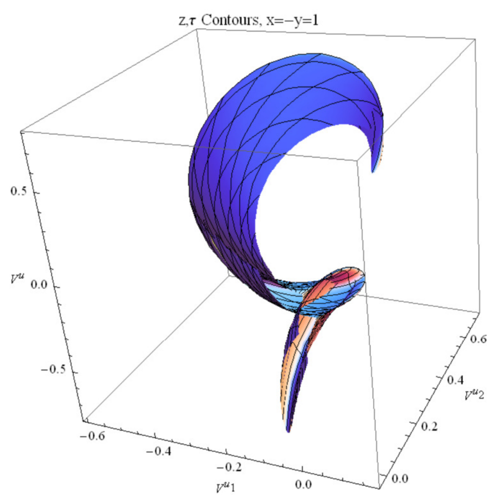


**Figure 11-25: Flow contours for attack-defense *free fall* with *harmonic steady-state wave* $\omega = 2 \quad x = -y = 1$**

7. In the player fixed frame model, the player support potential is a diagonal matrix in the space of the inactive strategies, or it is a diagonal matrix transformed by a constant rotation. Refer back to the field equations Table 4-5 for $\omega_{\nu\alpha\beta}$ and construct an argument for why this assumption might be broken if the co-moving frame orientation potentials were functions of proper time. Would it be necessary to re-compute the values of the orientation potentials?
8. So far we have considered locked behavior in which the pressure is a maximum at the *still point*. We now consider a different attack-defense model, one in which the payoffs are not scaled by the factor $\frac{1}{10}$, but are left with their initial values. We make changes to the self-support contributions

to insure that the pressure is initially zero. The basic changed values from the *free fall* model in sections 8.7 and 9.2 are given below. We get new behavior for the pressure (Figure 11-26) and similar behavior for the acceleration (Figure 11-27) in which the *still point* is the low pressure point $z \approx 0.02$. Because the payoff field carries energy, we are forced to consider a smaller lattice with sizes $\{sizeX \quad sizeY\}$ in the $\{x \quad y\}$ directions, respectively.

$$G_{1(blue)} = \begin{pmatrix} 4 & 1 \\ 3 & 4 \end{pmatrix}$$
$$G_{2(red)} = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$$
$$\omega_{z\alpha\beta} = \begin{pmatrix} 4 & 0 & 2.90363... \\ 0 & 5 & -0.725908... \\ 2.90363... & -0.725908... & 6 \end{pmatrix} \tag{11.3}$$
$$sizeX = sizeY = 1/5$$
$$-zLow = zHigh = 1/20$$
$$playerBias_1 = playerBias_2 = 0$$

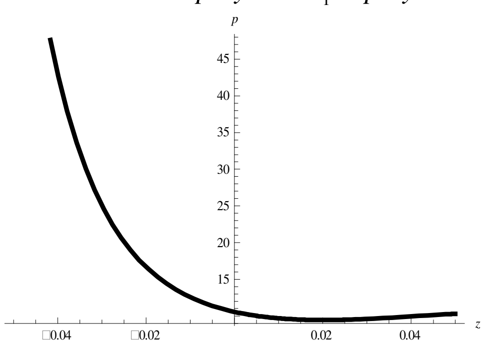


**Figure 11-26: Attack-defense model pressure with modest payoffs**

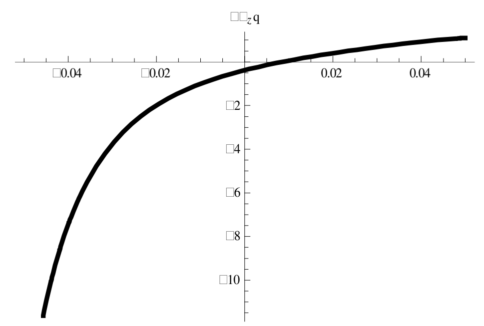


**Figure 11-27: Attack-defense model acceleration with modest payoffs**

9. For the attack-defense model, exercise 8, the lack of dominating player bias fields leads to more structure in the player interest for "blue" (Figure 11-28) and "red" (Figure 11-29). Is this structure what you would expect?

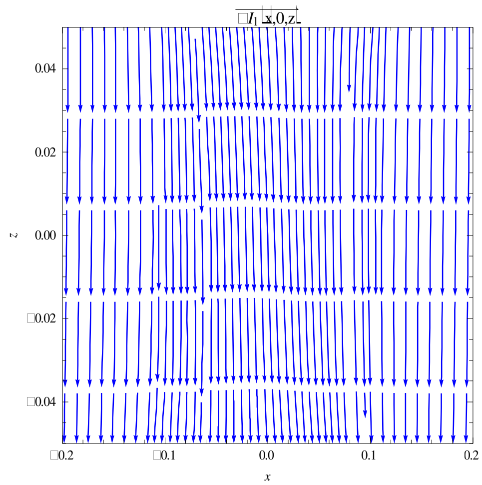


**Figure 11-28: Attack-defense model player interest for "Blue"**

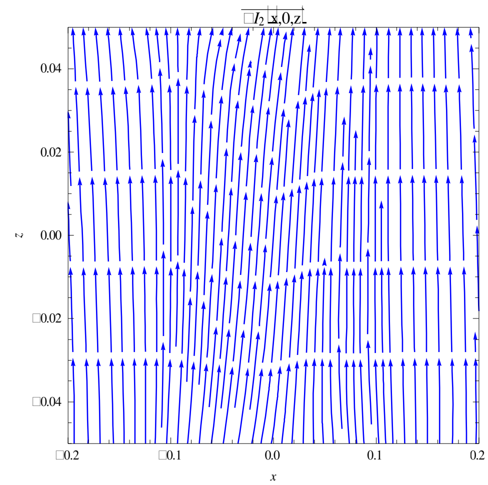


**Figure 11-29: Attack-defense model player interest for "Red"**

10. For the attack-defense model, exercise 8, there is now more structure in the acceleration gradients (Figure 11-30) and in the compression gradients (Figure 11-32). Explain the shape of the pressure curve Figure 11-31. Note that the minimum as seen along the $z$ contour is a saddle shape reflecting a maximum as seen along the $x$ contour. A similar shape is given for the pressure at $x = 0$. Given that the shape along $x$ and $y$ directions, can you see that there are multiple pressure ridges along constant $x$ directions. Additional insight can be gleaned from the *absolute acceleration* Figure 11-33. Using the ideas of ***structural stability*** from the one-dimensional model of the prisoner's dilemma, section 5.6, we see that a stable curve is not along the $x$ or $z$ axis, but a curve that accommodates the saddle points. Again similar curves exist for $x = 0$. What changes would you anticipate if the payoff for player 2 were to increase or decrease? Carry out the calculations to see if your expectations are met.

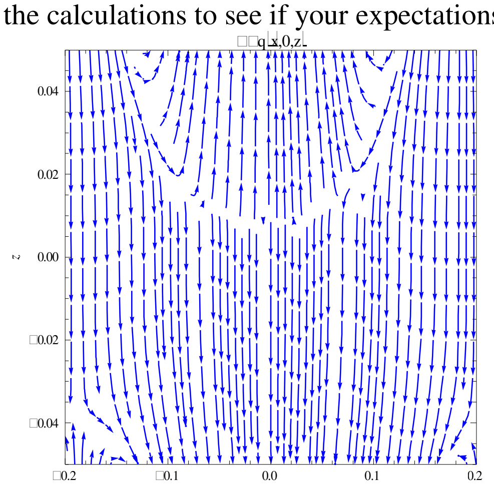


**Figure 11-30: Attack-defense model acceleration gradients**

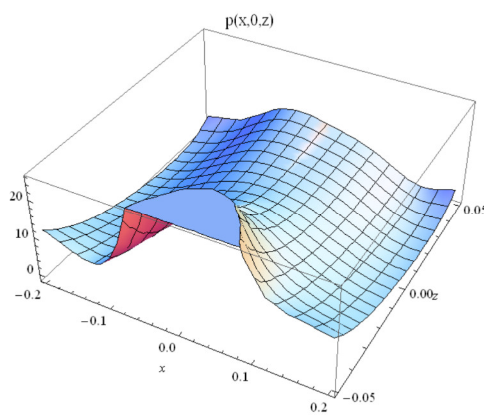


**Figure 11-31: Attack-defense model pressure for $y = 0$**

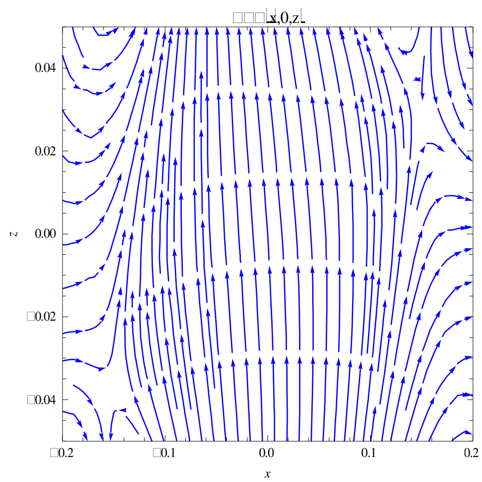


**Figure 11-32: Attack-defense model compression gradients**

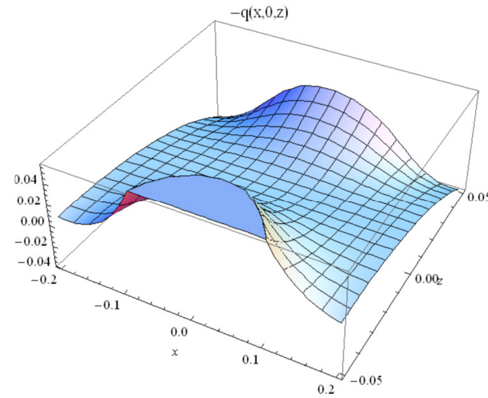


**Figure 11-33: Attack-defense model absolute acceleration for** $y = 0$