6 Robinson Crusoe economics

In the last chapter we developed the prisoner's dilemma scenario with a single active strategy and three codes of conduct. We showed the importance of codes of conduct in addition to choices of strategy. We didn't consider the simplest model however. In this chapter we consider what we think is the simplest game, one with a single active strategy and no codes of conduct, the Robinson Crusoe scenario.

To put the choice of this scenario in context, we identify a systematic set of steps for discussing decision processes. Part of a systematic approach is to have a taxonomy for decision processes, which plays a central role in the theory of games (Von Neumann & Morgenstern, 1944). Games can be zero-sum or non-zero sum; they can have multiple players starting from one to any number. The players can cooperate or choose not to cooperate. These distinctions have led to a focus in the literature on specific aspects of games based on a detailed taxonomy.

Decision process theory shares some of the same concepts of game theory, while introducing a fairly large additional set of concepts. The game theory taxonomy needs to be extended. In this chapter we start with a discussion of the types of changes needed (section 6.1). This leads naturally to a discussion of how to categorize the different possible decision processes; we take that up first and discuss a taxonomy or organization for the theory. The discussion leads naturally to defining a systematic approach for analyzing decision processes. We provide a provisional approach and then follow that approach for the Robinson Crusoe model (sections 6.2-6.8).

6.1 *Decision process taxonomy*

As an example of aspects of *decision process theory* that are extensions of game theory, we have established that decision processes exhibit stresses and strains, which help characterize dynamic behaviors. Another extension is the idea of a *code of conduct*. We have illustrated the importance of setting the *code of conduct*, noting it has profound influence over the dynamic behaviors of decision processes.

There are many possibilities of how these concepts inter-relate in realistic games, so it would be useful to have a systematic program of identification for all decision processes and a plan for applying *decision process theory* to each. We make a start by suggesting the components for such a systematic approach to and taxonomy of, decision processes. This puts into context the prisoner's dilemma from the last chapter, the Robinson Crusoe model in this chapter, and a general set of two person games in chapters 8 and 11.

Though we have yet to formally establish a systematic program of identification, we have identified the practical steps needed to solve problems at every level, at least for quasi-stationary solutions (those analogous to AC circuits). The steps are:

- Identify the story behind the decision process.
- Reframe the story so that it is in ***normal form***.
- Identify the ***natural units***.
- Transform to the ***co-moving coordinate basis***
- Use the *player fixed frame model (or a model that satisfies the quasi-stationary hypothesis)* as an initial model and find the *co-moving frame* solutions, which are *stationary*. Apply numerical methods to find these solutions. We have carried out this program in Chapters 8 and 11 with more than one active strategy using *Mathematica*© (Wolfram, 1992) along with standard techniques that have been used in other areas of applied mathematics and engineering, (Courant & Hilbert, 1962).

- Apply *decision process theory* to identify stable or *stationary* behaviors that serve as *known (boundary) conditions*. Use these behaviors to create the *stationary* flow streamline solutions from section 4.5.
- Add to the *stationary* streamline solution the *harmonic* streamline solutions from section 4.5, from which one can build an understanding of the system response to stimuli and construct solutions from any given set of initial time conditions.

Based on this process, a ***natural taxonomy*** of decision process extends the usual game theory taxonomy by considering the number a of active decisions: one active, two active, etc. Behind this decomposition will be the number i of inactive decisions that reflects both the number of players and actual number of strategies. In this decomposition, time may be active or inactive. In decomposing the decision process into active and inactive strategies, we take into account any operative *codes of conduct*. Processes are classified as the ordered pair (d_a, d_i) we call the ***strategic decomposition***. So for example in the prisoner's dilemma of the previous chapter, there are two players each with two strategies. We treated this as a single active strategy model with five inactive strategies including the two associated with each player, the ordered pair $(1,5)$.

An ***irreducible decision process*** is one in which each player has exactly one inactive strategy. We call a player ***dependent*** if the inactive strategy is their only strategy. Thus for an *irreducible decision process*, the number of players is equal to d_i, the number of inactive strategies. The possible decision processes are then associated with the ways in which we can partition the d_a active strategies to each player, including the possibility that a player has no active strategies and is *dependent*. We split the number d_i of inactive strategies into those associated with active strategies $d_{i\searrow a}$ and those that are dependent $d_{i\searrow d}$. Thus a further breakdown in our classification (d_a, d_i) is $(d_a, d_{i\searrow a} : d_{i\searrow d})$, which explicitly identifies the number of players $d_{i\searrow d}$ that are *dependent* and the number of players $d_{i\searrow a}$ that make one or more strategic choice.

In section 4.6 we introduced the *code of conduct* as a ***symmetric decision process*** in which each inactive strategy that is part of the *code of conduct* is associated with a new distinct player. Thus we start with the most general solution of the prisoner's dilemma in the *normal-form coordinate basis* and consider four active and two inactive strategies, $(4, 2:0)$. There are three ways to partition 4 strategies to 2 players. For the prisoner's dilemma, we pick 2 strategies to each player. When expressing the *known behaviors*, we connect the strategy flows and payoffs with the corresponding flows and metric potentials in the natural holonomic basis. We can then transform these values to the *symmetric normal-form coordinate basis* in which the code of conduct attributes are exhibited as new players. We classify the symmetric model and identify its place in the taxonomy. So for the prisoner's dilemma we end up with the ordered pair $(1,1:4)$.

It is not uncommon when discussing taxonomies (Von Neumann & Morgenstern, 1944) to propose a general program for solving each general class and in that way build up support for the view that the theory has general applicability. We have not progressed yet to that stage. We have provided a single model so far and have identified some general principles. We schedule further progress on this program to future work. In this chapter, we take a tentative step and work out the details of the simplest single strategy decision process that has a single inactive strategy corresponding to the strategic decomposition $(1,1:0)$. This example is useful as it illustrates the systematic approach required for the general case. The example is based on the Robinson Crusoe example discussed in (Von Neumann & Morgenstern, 1944, pp. 9, 15, 31, 87, 555). In chapters 8 and 11, we extend this approach to what would be called two-person games in game theory.

6.2 *Identify the story—Robinson Crusoe*

The Robinson Crusoe scenario is based on the strategic decomposition of a single active and a single inactive strategy, $(1,1:0)$. In *game theory*, we imagine a single player with a single choice to be made. The solution to this situation is thought to be one in which the player simply optimizes his or her choice. It might be thought that the game has little strategic interest. We disagree. We consider that case in ***decision process theory***, keeping an open mind however, about whether the game demonstrates behaviors of interest. As part of keeping an open mind, it is valuable and instructive to revisit the story on which this example rests. Note that we might not be as sympathetic today towards Robinson Crusoe as earlier readers.

The usual economic interpretation is based on the story by Daniel Defoe in which Robinson Crusoe is stranded on an island. Before he meets and rescues Friday, he survives through his "own efforts" of hunting and farming. Here's the plot summary from Wikipedia, 2008:

> "Crusoe leaves England setting sail from the Queens Dock in Hull on a sea voyage in September, 1651, against the wishes of his parents. After a tumultuous journey that sees his ship wrecked by a vicious storm, his lust for the sea remains so strong that he sets out to sea again. This journey too ends in disaster as the ship is taken over by Salé pirates and Crusoe becomes the slave of a Moor. He manages to escape with a boat and is befriended by the Captain of a Portuguese ship off the western coast of Africa. The ship is en route to Brazil. There with the help of the captain, Crusoe becomes owner of a plantation.
>
> "He joins an expedition to bring slaves from Africa, but he is shipwrecked in a storm about forty miles out to sea on an island near the mouth of the Orinoco River on September 30, 1659. His companions all die; he fetches arms, tools and other supplies from the ship before it breaks apart and sinks. He then gets battered by huge waves as he struggles to make it to an unknown island. He proceeds to build a fenced-in habitation and cave. He keeps a calendar by making marks in a wooden cross he builds. He hunts, grows corn, learns to make pottery, raises goats, etc. He reads the Bible and suddenly becomes religious, thanking God for his fate in which nothing is missing but society.
>
> "He discovers native cannibals occasionally visit the island to kill and eat prisoners. At first he plans to kill the savages for their abomination, but then he realizes that he has no right to do so as the cannibals have not attacked him and do not knowingly commit a crime. He dreams of capturing one or two servants by freeing some prisoners and indeed, when a prisoner manages to escape, Crusoe helps him, naming his new companion "Friday" after the day of the week he appeared and teaches him English and converts him to Christianity.
>
> "After another party of natives arrives to partake in a grisly feast, Crusoe and Friday manage to kill most of the natives and save two of the prisoners. One is Friday's father and the other is a Spaniard, who informs Crusoe that there are other Spaniards shipwrecked on the mainland. A plan is devised where the Spaniard would return with Friday's father to the mainland and bring back the others, build a ship and sail to a Spanish port.
>
> "Before the Spaniards return, an English ship appears; mutineers have taken control of the ship and intend to maroon their former captain on the island. The captain and Crusoe manage to retake the ship. They leave for England, leaving behind three of the mutineers to fend for themselves and inform the Spaniards what happened. Crusoe leaves the island on December 19, 1686."

In the story, Robinson Crusoe is evidently not alone since he has the help of Friday and others. Moreover, the outcome for him is dictated as much by the morality of the times as the economic situation. His ***worldview*** and in particular his ***self*** or independence, plays a key role and must be considered along with the payoff aspects of his decision processes. This *simplest* economic situation is therefore one in which there is a single agent and two strategies: a persistent internal inactive self-strategy associated with the ***player's interest flow*** and an active external active strategy that through the player's actions, produces outcomes.

We characterize the Robinson Crusoe example as one in which a subject forages (hunts and farms), whose sole strategy is to determine how much or how little of this he or she should do. The assumption is that there are sufficiently abundant resources so that no choice need be made on which approach to take (hunt or farm). It makes sense that anybody *stranded* on a desert island starts out with significant prior experiences. Just as in the Robinson Crusoe case, a subject brings with them a set of fundamental beliefs, which is their worldview (*Cf.* section 7.2). Someone brought up to be interdependent might be more likely to conserve even with super abundant resources; they might save for a rainy day or a day in which someone else might be stranded. They might treat Friday better. Someone brought up to be independent and a little self-centered might strive only to optimize his or her own pleasure. Thus there may in fact be behaviors of strategic interest even in this *simple* example. We show how *decision process theory* handles such possibilities, starting in the next section where we put this story into normal form.

6.3 *Put story into normal form*

Somebody stranded on an island will make many decisions during the course of the day. Our ordinary way of thinking about decisions is a series of sequential actions in time. We normally don't think through all of the possible consequences that flow from each single decision in time. It is like a chess game in which we consider each move based on what the opponent has done; on how the board appears. However, we can make our plans based on our first move, considering each possible move by the opponent and what our counter move would be and so on. We would create a very long list of possibilities, which we have called *pure strategies* in *normal form*. Our opponent, if there is one, can also make such a list and would thus create the opposing team's set of pure strategies. The play of the game of chess would be a single choice by us and a single choice by our opponent. This is called putting the process into normal form. For convenience we label each string of choices with an informative name, which should not be confused with the myriad individual choices that need to be made to actually carry out the decision process.

With that in mind we characterize the person stranded on the island as having a single pure strategic choice to make, which we characterize by ***foraging***, which we hope is an informative name. It represents possibly hundreds of individual decisions during the day, all of which are characterized by this name. The degree to which foraging occurs is indicated by the value of a single ***active strategy*** variable, u. Our convention is that positive values are indicative of actively foraging and negative values are indicative of restraint or resistance to foraging (conserving).

In addition to the activity of foraging, there is the mindset or worldview of the person stranded on the island, which we characterize by a single ***inactive strategy*** variable w. We begin with the idea that a positive value indicates an internal belief in the value of getting as much off the island as possible, whereas a negative value indicates a desire to conserve what is there and take only the minimal necessary to insure personal survival. The normal form has a single active and single inactive strategy, $(1,1)$, corresponding to a 3-dimensional space-time. There is a single payoff field with non-zero component $F^{w}{}_{tu}$; there are no purely spatial components of the payoff and so no payoffs that correspond to ordinary *game theory*. There is only the *game value* represented by this time component, which we have also called the *electric field* component. We use the single strategy model and streamline solutions from section 4.6.

6.4 *Identify the natural units*

We can think of time as being marked off in days, with each day a complete play of the strategy. This incorporates a complete play. The natural velocity is the effort Robinson Crusoe can do in one day. Utility is measured in the same one-day equivalences. In the next section we analyze this model in the *co-moving coordinate basis*.

6.5 *Co-moving coordinate basis*

The *co-moving frame* flow equations, Table 4-1, are expressed in terms of the flow direction o, the *proper inactive strategy* direction ξ and the *proper active* direction υ. The flow equations depend on the acceleration $q^{\upsilon}=-q_{\upsilon}$, a single vorticity $\omega_{\upsilon\xi}\equiv\omega_{\upsilon}$, strategy υ, compression coefficient $\omega_{\upsilon\xi\xi}\equiv-\Theta_{\upsilon}=\Theta^{\upsilon}$ and no shear components, Eq. (4.107):

$$
\begin{aligned}
&\partial_{\upsilon}E_{w\xi}=\Theta^{\upsilon}E_{w\xi}\\
&\partial_{\upsilon}e_{\xi}=2\omega_{\upsilon}-\left(q_{\upsilon}-\Theta_{\upsilon}\right)e_{\xi}\\
&\partial_{\upsilon}E_{wo}=-q_{\upsilon}E_{wo}+2\omega_{\upsilon}E_{w\xi}\Leftrightarrow E_{wo}=E_{w\xi}e_{\xi}\\
&f^{w}{}_{o\upsilon}=2\gamma^{ww}E_{w\xi}\left(q_{\upsilon}e_{\xi}-\omega_{\upsilon}\left(1+e_{\xi}e_{\xi}\right)-\Theta_{\upsilon}e_{\xi}\right)\left(1-\gamma^{ww}E_{wo}E_{wo}\right)\\
&\gamma_{ww}=E_{wo}E_{wo}-E_{w\xi}E_{w\xi}=-E_{w\xi}E_{w\xi}\left(1-e_{\xi}e_{\xi}\right)
\end{aligned}
\tag{6.1}
$$

The acceleration, compression , vorticity and pressure are determined from the tidal, electromagnetic and inertial equations from chapter 4, Eq. (4.111), (4.109), (4.108) and (4.110):

$$
\begin{aligned}
&\partial_\upsilon q_\upsilon = {q_\upsilon}^2 - \Theta^\upsilon q_\upsilon + 2{\omega_\upsilon}^2 - 2\kappa p \\
&\partial_\upsilon \omega_\upsilon = 2\omega_\upsilon q_\upsilon \\
&\partial_\upsilon \Theta^\upsilon = 3{\omega_\upsilon}^2 - {\Theta_\upsilon}^2 - \kappa\mu \\
&\kappa {p^\upsilon}_\upsilon = -\Theta^\upsilon q_\upsilon + {\omega_\upsilon}^2 \\
&p_{\upsilon\xi} = 0
\end{aligned}
\tag{6.2}
$$

The inactive pressure $p_{\xi\xi}$ and energy density are undetermined. For this investigation we pick the following:

$$
\begin{aligned}
&{p^\upsilon}_\upsilon = {p^\xi}_\xi = p \\
&\mu = \alpha p
\end{aligned}
\tag{6.3}
$$

As before (section 5.5.4), we call the parameter α the resiliency.

In addition to the *stationary* equations, we have the time dependent partial differential equation Eq. (4.112):

$$\left(1 - {e_\xi}^2 - {q_\upsilon}^2\tau^2\right){\partial_\tau}^2 x^a - {\partial_\upsilon}^2 x^a + 2\tau q_\upsilon \partial_\upsilon \partial_\tau x^a + q_\upsilon\left(q^\upsilon + 2\Theta^\upsilon\right)\varpi\partial_\tau x^a - \left(q^\upsilon + \Theta^\upsilon\right)\partial_\upsilon x^a = 0 \tag{6.4}$$

As the *known behaviors*, we start with zero flow along the active strategy and zero flow along the inactive strategy. We look to the equations to see whether this is maintained. As in the prisoner's dilemma model, we solve these equations numerically using the software *Mathematica*, (Wolfram, 1992). We take up the known behaviors in more detail in the next section.

6.6 ***Known behaviors***

The essential aspects of the solution depend on Eq. (6.1), (6.2) and (6.4), with appropriate known behaviors. As in the prisoner's dilemma, we take these known values for the differential equations to be when the active parameter $\upsilon = 0$. Note however that in this case the active strategy variable is a measure of how much or how little foraging will be done. There is freedom to choose the ***co-moving frame*** for the flows as mentioned above. This is the frame, in which the flow is along the time direction so the space components vanish, $V^w = V^\upsilon = 0$. In this frame, the time component of flow $V^0 = e^{-\Phi}$ defines the *gravitational potential* Φ.

The vorticity parameter ω_υ is related to the electric field, Eq. (6.1):

$$f^w{}_{o\upsilon} = 2\gamma^{ww} E_{w\xi}\left(q_\upsilon e_\xi - \omega_\upsilon\left(1 + e_\xi e_\xi\right) + \Theta^\upsilon e_\xi\right)\left(1 - \gamma^{ww} E_{wo} E_{wo}\right) \tag{6.5}$$

Since the vorticity is a rotation in the $\upsilon - \xi$ plane, we can use this equation to understand somewhat better the relationship between the ***self-***direc***tion*** and the active strategy direction.

The known values of the transformations at the origin are taken so that the *co-moving basis* and *normal-form coordinate basis* are aligned at the origin $\upsilon = 0$:

$$
\begin{aligned}
&{E^u}_\upsilon = {E^w}_\xi = 1 \\
&{E^u}_\xi = e_\xi {E^u}_o = {E^w}_\upsilon = 0
\end{aligned}
\tag{6.6}
$$

If we take other values for the known flow at the origin, then we use the orthogonalization process described in section 5.5 to determine these transformations.

The remaining scalar acceleration, stress (pressure) and strain (compression) $\{q_\upsilon \quad p \quad \Theta^\upsilon\}$ are set to represent a solution that has a small force towards more foraging, a small stress and a strain Θ^υ that yields a negative vorticity from Eq. (6.2):

$$\begin{aligned}&\{q_\upsilon \quad p \quad \Theta^\upsilon\} = \{-0.01 \quad 0.01 \quad -2\}\\&\omega_\upsilon = -0.1732...\end{aligned} \tag{6.7}$$

The units are $\kappa = 1$ and the ***resilience*** $\alpha = 5$. It may be noted that these values are not dissimilar to those used in the prisoner's dilemma for the sensitivity analysis. In the next section we consider the consequences of these known behaviors on the scalar behaviors that are *stationary*.

6.7 *Stationary behaviors*

For the prisoner's dilemma, our known behavior was for zero acceleration and a maximum pressure. However we did find in our sensitivity analysis, solutions that had zero acceleration and a minimum pressure. Based on the known behaviors from the previous section, we get a similar result for the stress (pressure) goes to a minimum, Figure 6-1 and the acceleration goes to zero, Figure 6-2.

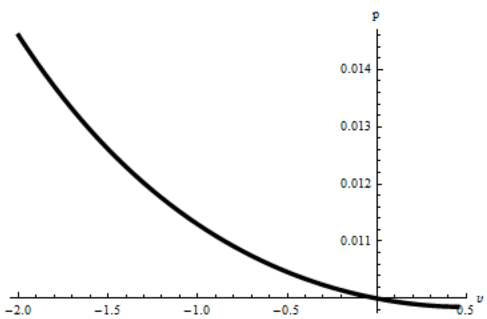

Figure 6-1: Stress, $p(\upsilon)$

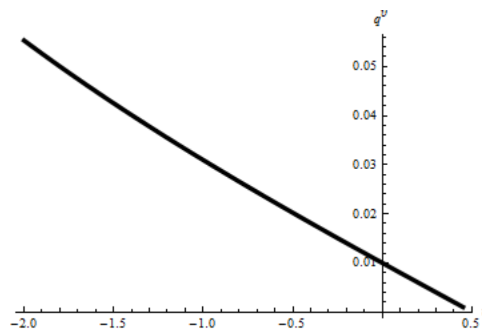

Figure 6-2: Acceleration, $q^\upsilon(\upsilon) = -q_\upsilon(\upsilon)$

The solutions are different in that the space is finite and gets progressively smaller as we increase foraging, Figure 6-3. Because the compression drops so precipitously, the volume element of space goes to zero. Here as in the figures for the acceleration, the value never quite gets to zero. The acceleration change is balanced by the vorticity, Figure 6-4, which levels off. The net effect seems to be consistent with a flow that goes to the right but progressively slows down and never passes a certain boundary or size.

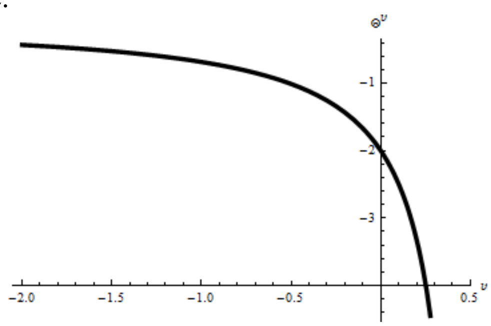

Figure 6-3: Compression, $\Theta^\upsilon(\upsilon)$

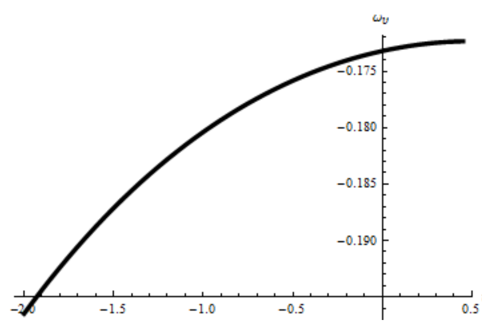

Figure 6-4: Vorticity, $\omega_\upsilon(\upsilon)$

A way to appreciate what is happening is to consider the proper charge, Figure 6-5. The charge is positive as we move to the right and negative as we move to the left. It might be that the positivity of charge reflects more independence. It appears along with a tendency towards foraging. A more direct measure is the Killing vector along the inactive direction. One component is the transformation $E_w^{\ \xi}$, which has the known value of unity at the origin and decreases to zero as we go towards more *foraging*, Figure 6-6. From this we obtain the charge $E_w^{\ o} = e^\xi E_w^{\ \xi}$, Eq. (6.1).

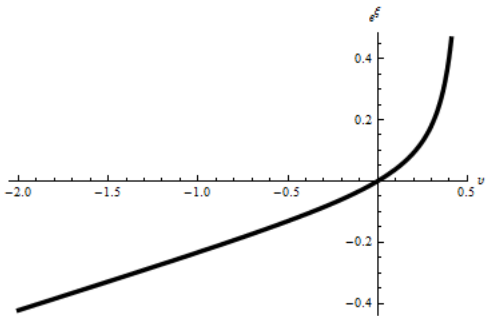

Figure 6-5: **Proper charge,** $e^{\xi}(\upsilon) = -e_{\xi}(\upsilon)$

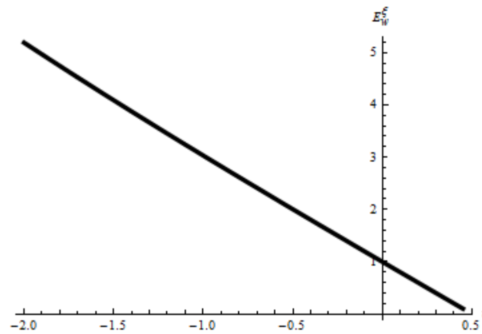

Figure 6-6**: Killing component** $E_w{}^{\xi}(\upsilon)$

The charge has the same sign as the proper charge at the origin, Figure 6-7. We associate the value of the decision process with the time component of the field, which in the *co-moving frame* is $f^{w}{}_{ov}$, Figure 6-8. As the charge increases, we get an increasing decision value. This appears to reflect the same phenomena as in *game theory* for the one person game: maximizing the utility is the purpose of the single strategy. The more foraging that goes on, the more the activity is independent and the higher the *game value*.

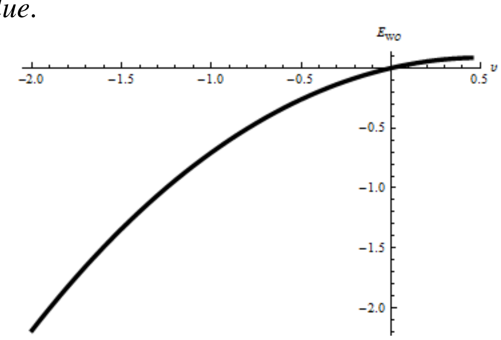

Figure 6-7: Inactive flow, $E_{wo}(\upsilon)$

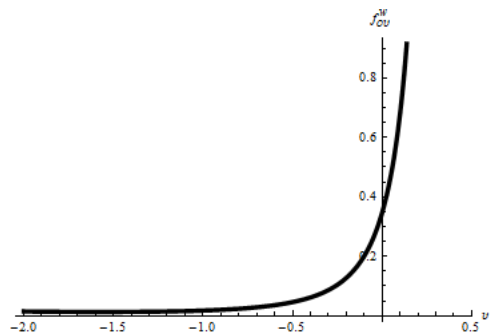

Figure 6-8: Decision value, $f^{w}{}_{ov}(\upsilon)$

There appears to be no value towards acting interdependently; as we move to the left, the value of the decision process stays and gets progressively smaller. It is important to note however that in our *decision process theory*, this situation represents only one of two possible solutions to Eq. (6.2). We can change the sign of both the acceleration and the strain and again obtain a solution to the equations keeping the same value of the vorticity. The decision value will again be positive, but will display its maximum on the left corresponding to a restraint on foraging and a tendency towards interdependence not independence.

Another observation is that there are solutions in which the decision value is negative, corresponding to picking the positive solution for vorticity. In this case the proper charge to the right of the origin becomes negative and to the left becomes positive. We have no interpretation for this at the moment. A tentative interpretation is that to the right, the environment is inhospitable. The more one interacts with it, the more likely it is that one will be hurt. In this case foraging is not such a good activity. The safest approach is to be more of a conservationist and hope to come out at zero value. But again there will be the opposite of this solution in which interacting with the hostile environment can come out to be the best approach and being interdependent is less successful.

Which of these choices is the best? In our *decision process theory*, all are equally possible. It is in this sense that the *decision process theory* provides a common ground. To determine what governs a given situation, a measure of the acceleration as well as the stresses and strains is expected to provide the determination. One way to see the effects of the stresses and strains is to consider the *harmonic* effects. In the next section we consider what we learn from a *harmonic* stimulus.

6.8 ***Harmonic behaviors***

In the prisoner's dilemma, section 5.9, we saw wave-like behaviors. We explore this in more detail in this section and conclude that there are no travelling *gravity waves* in the central holonomic frame but in the *normal coordinate frame* there are ***transient frame waves***. These transient waves die out based on the strengths of the stresses and strains. What we find in fact is evidence in the solutions of the theory for steady state wave phenomena analogous to AC circuit behaviors.

Einstein's theory of general relativity predicts that there will be ***gravity waves***; this is a property of all theories of this type. Ours is one of them. The number of independent components $\frac{1}{2}d\left(d-3\right)$ of these *gravity waves* depends on the dimension of space-time d. The wave components are those components that are left over from the total number of field variables (based on the number of independent metric components in a holonomic frame), $\frac{1}{2}d\left(d+1\right)$ minus the number of gauge conditions d and minus the number of field equations that are independent of time, which can be shown to be d based on the Codacci equations. On general grounds we see that for our specific case of $\left(1,1\right)$ active and inactive components that the dimension of space-time is $d=3$ and so there should be no *gravity wave* components. There will be *frame wave* components, which can be transformed away by a suitable coordinate (gauge) transformation.

For a general theory in which there are d_a+1 active strategies (including time) and d_i inactive strategies, the *gravitational waves* manifest in one of three ways: they are part of the active space of which there are $\frac{1}{2}\left(d_a+1\right)\left(d_a-2\right)$ components; they are part of the inactive space and so consist of the inactive metric components of which there are $\frac{1}{2}d_i\left(d_i+1\right)$ components; or they are part of the d_i payoff potential fields Eq. (1.21), of which there are d_a-1 components for each inactive strategy. The total number of components adds up to the same:

$$\frac{1}{2}\left(d_a+1\right)\left(d_a-2\right)+\frac{1}{2}d_i\left(d_i+1\right)+d_i\left(d_a-1\right)=\frac{1}{2}\left(d_a+d_i+1\right)\left(d_a+d_i-2\right)=\frac{1}{2}d\left(d-3\right) \quad (6.8)$$

In particular, for the *player fixed frame model* in the *co-moving frame*, we have eliminated the time behavior of the inactive metric as well as any possible wave behaviors for the payoff fields. We would not expect any *gravity waves* unless there are at least three or more active strategies. It may still happen that the model is sufficiently restrictive that all *gravity waves* are absent.

In the *central holonomic frame*, section 4.5.6, the active metric elements are independent of the *central time*, section 4.8, exercise 45. Thus even with three or more active strategies there will be no wave propagation of the metric elements in this frame. You might believe that a *gravity wave* should be present in every frame. However in a frame that moves with the media, we can't see these waves. We do see *gravity waves* in the *normal coordinate basis*, since we see the motion of the media and fluctuations in that motion that appear as waves. We see evidence of wave phenomena in the in the coordinate scalars x^a and the frame transformations $\left\{\bar{E}^a_{\bar{\upsilon}}, \quad \bar{E}^a_{\bar{\tau}}\right\}$, *Cf.* section 4.8, exercise 43. The initial specification of the frame transformation on a space-like surface propagates to all values central time as a wave at the speed of "light," the maximum speed of the transmission which would be the same as the speed of a *gravity wave*. In general, such ***transient frame waves*** are attenuated, section 4.8, exercise 59. For an observer in such a frame, the *transient frame waves* are real and observable.

A transmission line with loss is an example: we introduce a wave stimulus, which propagates as a wave. Because of the resistance in the line, the wave attenuates over distance and ultimately vanishes. The theory would predict that there are no true waves that propagate in a lossy line. We would observe the wave as a transient effect. The distance over which the transient effects persist measures the loss of the line. With this in mind, we look at the solutions for our simple model over large enough regions so that we might observe whether or not the waves are transient. For the numerical parameters used in the previous sections, we choose a time interval $(0,1.2)$, a strategic space interval $(-2,0.45)$ and keep sufficient polynomial terms to obtain reasonable accuracy. By experiment we find that we must keep polynomials in time of order 100.

The limitations of the solution are determined by where the inactive metric component $\gamma_{ww}(\upsilon)$, Eq. (6.1) becomes zero. The metric component is proportional to the ***boundary difference*** determined by the proper charge:

$$\psi = 1 - e_{\xi}^{2} \tag{6.9}$$

We term this the boundary difference since we see that it tends to zero at the boundaries of the space, Figure 6-9. The boundary difference is also related to the metric potential for the active time component:

$$g^{tt} = E^{t}{}_{o}\psi E^{t}{}_{o} - E^{t}{}_{u}E^{t}{}_{u} \tag{6.10}$$

Since we take the known behavior to be $E^{t}{}_{u} = 0$ and $E^{t}{}_{o} = 1$, we see the close connection, even if these conditions are relaxed and only approximate. At least initially this metric component is the inverse of $g_{tt} = e^{2\Phi}$ and so directly related to the gravitational potential Φ.

6.8.1 Single strategy harmonics

To provide a stimulus to the system, we provide a *harmonic* contribution to the flows, Figure 6-10. We pick the same values used in section 5.9, Eq. (5.62). The color code is that purple represents the time flow as a function of the *proper time* and blue represent the flow along the active strategy direction u.

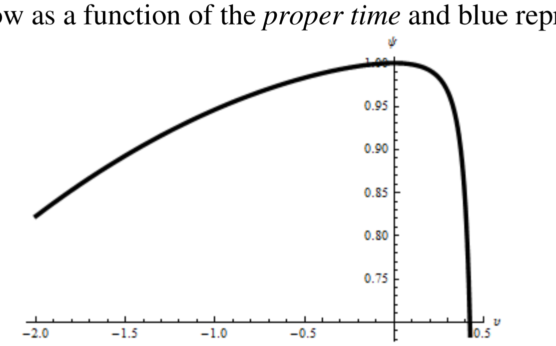

Figure 6-9: Boundary difference $\psi(\upsilon) = 1 - e_{\xi}^{2}$

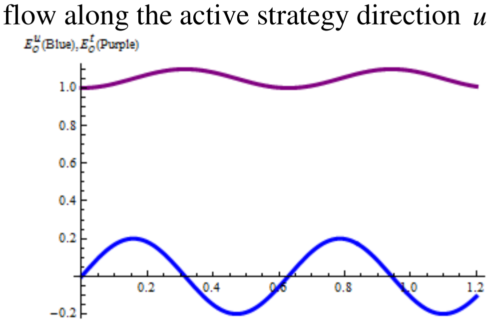

Figure 6-10: Strategic flows, $E^{a}{}_{o}(0,\tau)$

We are particularly interested in seeing whether the stimulus that we provide at the origin $\upsilon = 0$ travels outward in time and distance as a wave or as a transient wave. Our first look is at the contour plot of the coordinate transformation $E^{t}{}_{o}$, Figure 6-11. The oscillation structure is markedly smaller at large negative distances suggesting strongly that the wave is transient. It shows up clearly as well in the 3-dimensional view, Figure 6-12. The wave may damp because it comes to the boundary of the space or the boundary of the ***focus area***. The wave may also be reflected by the boundary. The energy of the system is conserved but may still be dissipated. We have seen that there are various forces at work: the pressure gradient, the payoff field that acts like an electric field and the gravity-like attraction field. The non-linear character of their interactions distorts the wave, so that its wave character becomes lost.

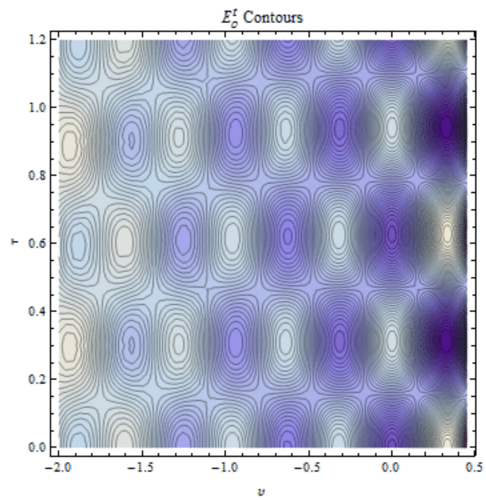

Figure 6-11: Time coordinate transformation contour plot $E^{t}{}_{o}\left(\upsilon,\tau\right)$

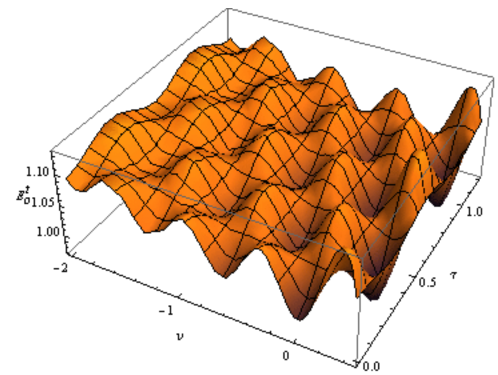

Figure 6-12: Time coordinate transformation $E^{t}{}_{o}\left(\upsilon,\tau\right)$

The active coordinates are ***harmonic coordinates*** and are expected to have wave like characteristics. It follows that we expect the same for the associated transformations. We have seen this for the time transformation.

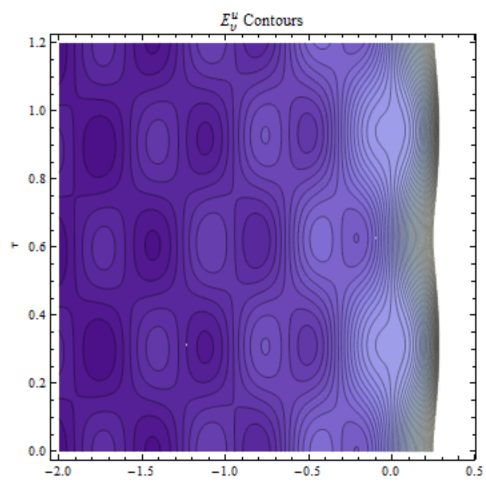

Figure 6-13: Space transformation contour plot $E^{u}{}_{\upsilon}\left(\upsilon,\tau\right)$

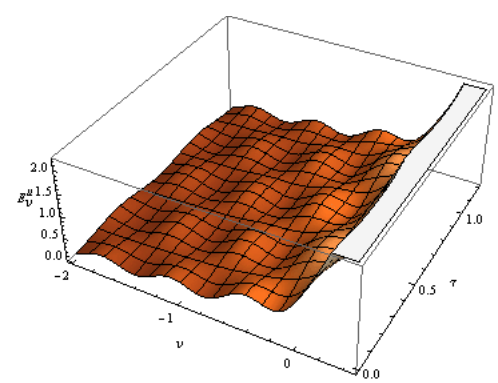

Figure 6-14: Space transformation $E^{u}{}_{\upsilon}\left(\upsilon,\tau\right)$

We now look, Figure 6-13, at the strategic space transformation, $E^{u}{}_{\upsilon}$, in which we have imposed a smaller *harmonic* contribution. This contour plot and the corresponding 3-dimensional plot, Figure 6-14 confirm the transient effect of the wave due to the initial stimulus.

6.8.2 Frame waves

The structure of the metric potentials in the *co-moving orthonormal frame* depends on transformations such as the strategic flow, Figure 6-15 and Figure 6-16. We see here and in the time flow Figure 6-11 that the ridges are not proceeding exactly along lines $t=\pm u$ that might be expected of waves. The patterns reflect a hexagonal packing and indicates that parts of the flow along the ridge indicates *stationary* behavior. These behaviors become folded together when computing the metric potentials.

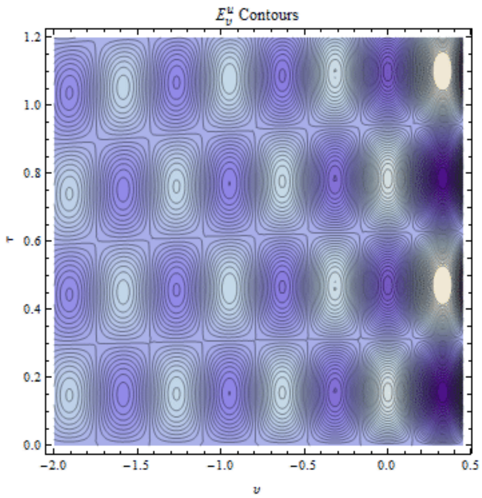

Figure 6-15: Strategic Flow contour plot $E^{u}{}_{o}(\upsilon,\tau)$

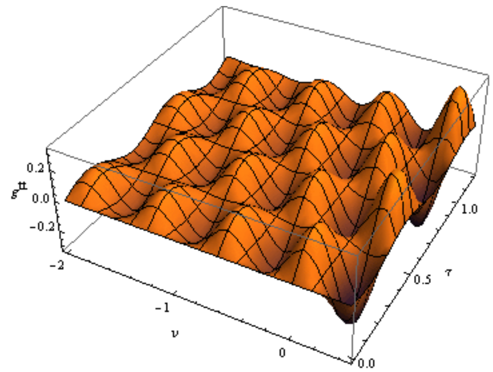

Figure 6-16: Strategic flow $E^{u}{}_{o}(\upsilon,\tau)$

Of particular interest is the time component of the metric g_{tt}, Figure 6-17 and Figure 6-18. The transient features we observed in the coordinate transformations persist here, along with the effects due to being near the boundary of space.

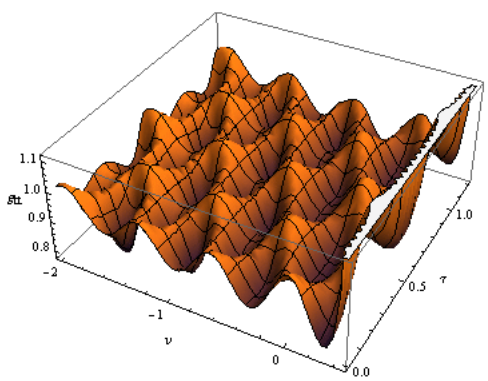

Figure 6-17: $g_{tt}(\upsilon,\tau)$

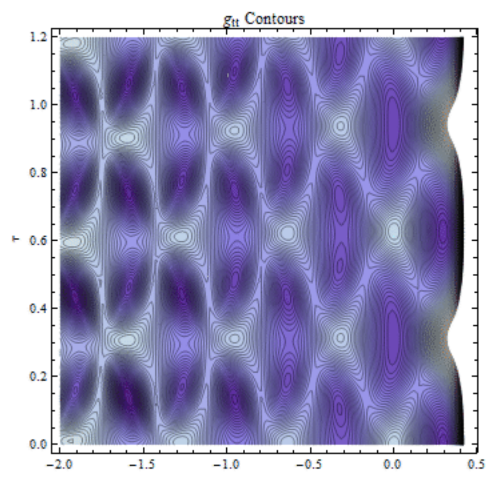

Figure 6-18: Contour plot of $g_{tt}(\upsilon,\tau)$

We see evidence of a new feature, which is that the wave pattern, observed around the origin, changes its character. The ridges become more *stationary*. Nevertheless, there is still a repetitive pattern, which should not be surprising since that was the nature of our known stimulus. We expect this aspect to disappear if we change the nature of the stimulus. For example to achieve a pulse at the origin of space and time, we add numerous *harmonics* together following standard techniques in Fourier analysis. The transient phenomena we have discussed for a single *harmonic* then carry over to the sum of many such *harmonics*.

6.8.3 Decision process values

In *decision process theory*, we replace the *game value* with the ***decision process value***, $F^{w}{}_{tu}(\upsilon,\tau)$. Based on the known behaviors, we start with Figure 6-19. We see that this wave is transient, Figure 6-20. *Decision process theory* provides the transients, which can be large or small depending on the nature of the stresses and strains.

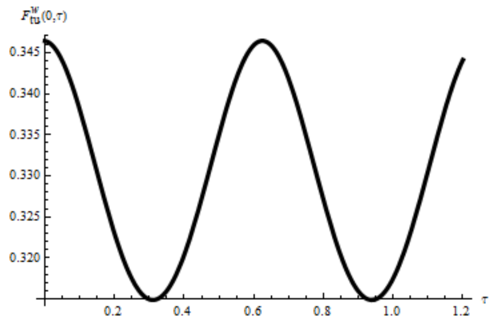

Figure 6-19: Known behavior $F^{w}{}_{tu}(0,\tau)$

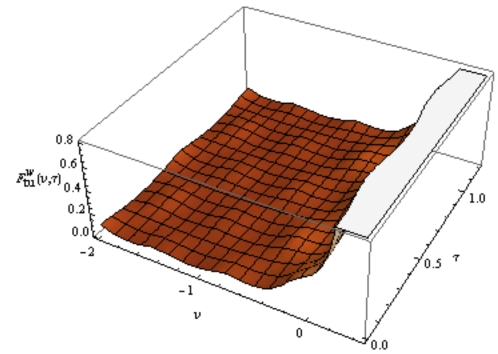

Figure 6-20: Decision value $F^{w}{}_{tu}(\upsilon,\tau)$

6.8.4 Transient frame waves

If we increase the size of the compression substantially, we narrow the ***focus area*** of the examples given above, Figure 6-21 and Figure 6-22. A dramatic "cloverleaf" structure is generated. Instead of the ridges moving along paths similar to $t=\pm u$, as in Figure 6-15, we see two ridges corresponding to constant time or constant distance. We speculate that we are seeing a new symmetry emerge from the equations, one that is generated from strong non-linear effects. We would not expect such behaviors extrapolating from weak field approximations.

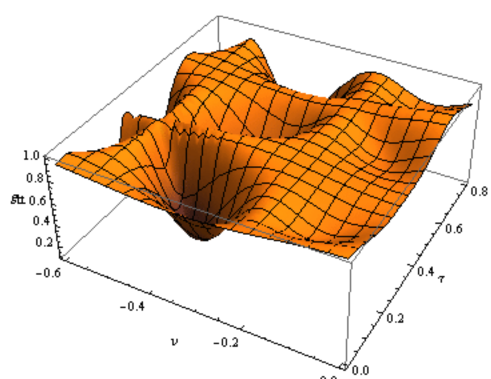

Figure 6-21: High compression $\Theta^{\upsilon}=-20$ **plot of metric potential** $g_{tt}(\upsilon,\tau)$

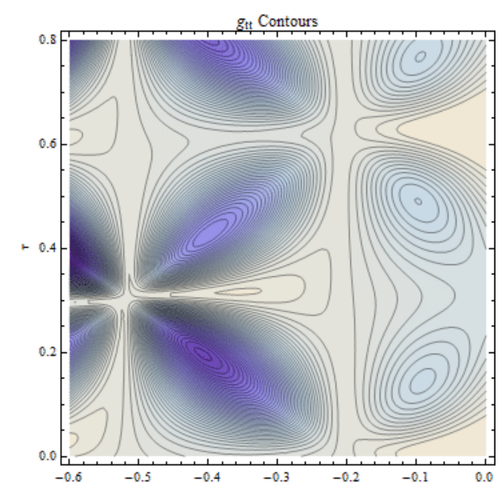

Figure 6-22: High compression $\Theta^{\upsilon}=-20$ **contour plot of metric potential** $g_{tt}(\upsilon,\tau)$

We also reduce the transient distance: the stimulus wave effects disappear more quickly. Though it is always difficult to prove theorems based purely on numerical examples, we believe that it is compelling that there are transient wave effects as well as new structures in the streamline solutions. The transient effects provide access to the underlying stresses and strains that we believe govern the dynamic behaviors of decision processes.

We thus draw important lessons from even this simple example with a single active and single inactive strategy. Non-linear effects may wash out wave phenomena but may create new structures. We see these structures reflected in other attributes of the solutions. See for example exercise 7.

6.9 ***Outcomes***

We have employed a general approach to decision process theory and used it to analyze a simple model of a subject who has a single active decision available and a single inactive strategy to study the interplay of stresses and strains. We argued that a study of the response to *harmonic* stimulus provides insight into the structure of these stresses and strains. Further, we provide support for the proposal (Thomas & Kane, 2008) that the inactive strategy is related to the *worldview* of the subject. The streamline solutions reflect the *harmonics*. They reflect an important attribute that if we collect matter or charge in a confined space, the matter generates pressure and the charge generates an electric field. These provide forces that inhibit more matter or like-charges from collecting and the result will be to create new dynamic structures. These conclusions are ones that can be applied to the general case.

The attainment of the outcomes of this chapter is facilitated by doing the exercises in the following section. Based on this investment, the student should achieve the more detailed outcomes below based on section.

- From section 6.1 understand how the game theory taxonomy extends to decision process theory.
- From section 6.2, the student should learn the importance of clearly articulating the underlying story or premise of the decision process.
- From section 6.3, the student understands the value of reframing the sequential story into a decision process in normal form.
- From section 6.4, the student learns that the natural units set the utility, time and action scales for the decision process.
- From section 6.5, the student learns the value of transforming the problem from the (symmetric) *normal-form coordinate basis* to the *co-moving coordinate basis*. The problem has the same content in the theory; each frame provides valuable insight into the problem.
- From section 6.6, the student must identify all the variables that must be initially specified and provide known values for these variables.
- From section 6.7, the student isolates the *stationary* behaviors of the problem, whereas in section 6.8 the student isolates the dynamic behaviors based on the wave nature of the solution using *harmonics* to build up any set of known conditions.

6.10 ***Exercises***

1. Because of the structure of the field equations of *decision process theory*, distinctly different games are characterized by the *strategic decomposition* $(a, i_a : i_d)$. We expect processes to reflect increasing complexity as we change the number of active strategies. So we see the progression as $(1,1:i_d)$, $(2,1:i_d),(2,2:i_d)$, $(3,1:i_d),(3,2:i_d),(3,3:i_d)$, *etc*. We have a fairly complete solution for processes of the form $(1,1:i_d)$, along with two examples: the prisoner's dilemma and the Robinson Crusoe example. What examples can you think of that illustrate game with more active strategies?
2. The prisoner's dilemma has solutions with a strategic decomposition of $(2,2:2)$ if we define the *code of conduct* such that the two strategies to confess, $(C_1 \quad C_2)$ are inactive and dependent. Describe the effects that will be new and distinct for such solutions. Show that in the co-moving frame there will now be a non-zero stationary *magnetic field*.

3. In *decision process theory*, to each player or agent there is allocated one inactive strategy. It thus makes sense that such strategies are not dynamically changed, but have values that are conserved. We say a player is an ***inactive player*** if every active strategy it is accountable for is treated inactive by all players in the decision process. This is an extreme example of an isometry of the solution. Discuss the equivalent decision process that has one fewer players. In particular discuss the effects the *inactive player* still has on the process.
4. Imagine a political game in which there are three "players": there is a player that chooses the political party (say A or B); there is the player that chooses a financial payoff to the party in power (say high or low); and there is the player that chooses the popular vote (say for A or for B). Assuming the game has been defined already in normal form, discuss the attributes of the game and suggest one or more possible *codes of conduct* and the form the equivalent game might take.
5. With the numerical calculation based on section 6.8.1, describe the effects seen in the holonomic coordinates, Figure 6-23.
6. How do the effects seen in Figure 6-23 compare to those in the prisoner's dilemma, section 5.9?

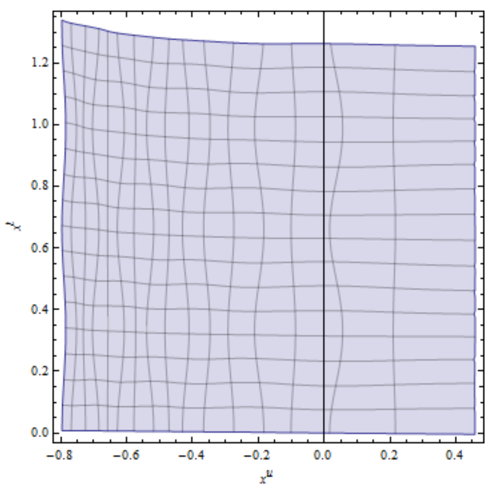

Figure 6-23: distance $x^u(\upsilon,\tau)$ versus time $x^t(\upsilon,\tau)$ for constant (υ,τ) contours

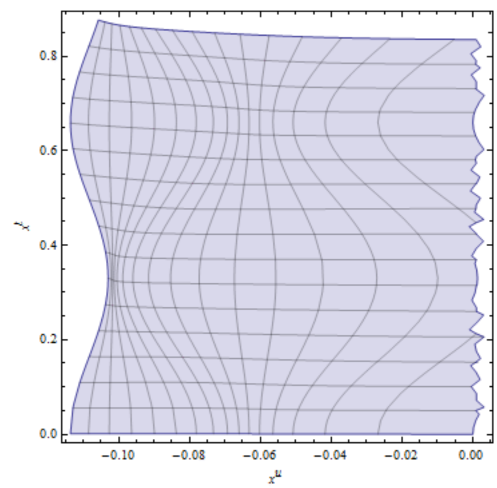

Figure 6-24: distance $x^u(\upsilon,\tau)$ versus time $x^t(\upsilon,\tau)$ for constant (υ,τ) contours with high compression $\Theta^\upsilon = -20$

7. With the numerical calculation based on section 6.8.4, describe the effects seen in the holonomic coordinates, Figure 6-24, for the case corresponding to high compression.