7 Process formulation

In this chapter, we sketch a general review of game theory and economic approaches to provide a further basis for the arguments provided in the previous chapters based on two rather simplified models. Though the models are simple, the analysis was based on a detailed and complete *decision process theory* from which one can study any decision process. We suggest that this general review provides further support for the theory.

We may ask whether we have sufficient information, insight and understanding to apply this theory to real world applications. For example we have analyzed the Prisoner's Dilemma in some detail (chapter 5). We noted that the Prisoner's Dilemma is a real paradox (section 5.6) for *game theory*. The Nash equilibrium for this game corresponds to the prisoners choosing options which are not in their public-interest. They both confess, which provides each a worse outcome than if they remained silent. The value of a paradox is that it forces us to ask "what is missing?" Based on that inquiry, we expanded our understanding. Since we are able to describe situations in which the prisoners don't confess, we have resolved the dilemma and have added something in our theory that is absent in *game theory*. We have not yet fully articulated these additions. In this section, we explore in more depth the concepts of *game theory* with the goal in mind of bringing forth such additions that are essentially conceptual issues. We base this in part on a selected review of the literature.

7.1 *Historical review*

Operationally, since our *decision process theory* goes beyond static or equilibrium behaviors, we know our solutions diverge from the literature. We differ markedly from the original treatment of (Von Neumann & Morgenstern, 1944) as well as from the extensions of (Nash, 1951). Our taxonomy in chapter 6 identifies many possible solutions that are not present in game theory, though this collection only scratches the surface of the solutions to the field equations presented in Eq. (2.31). The taxonomy (section 6.1) provides an enumeration and detailing of the steps needed in order to apply our *decision process theory* to practical situations and helps provide a clearer relationship of our *decision process theory* to *game theory*.

However, much of the enumeration and detailing can be taken from the existing literature. An examination of the literature is therefore essential, even though it won't necessarily help to identify conceptual differences between our *decision process theory* and *game theory*. To get at the conceptual issues, the historical context of *game theory* and its relationship to the *decision process theory* is explored. We start with an inquiry into both the context of *game theory* and earlier contexts that in some cases relate more directly to the approach we have taken with *decision process theory*.

We start with (Von Neumann & Morgenstern, 1944). This classic work spawned what is considered ***game theory*** and a significant amount of subsequent work, which is well reviewed in the text by (Luce & Raiffa, 1957). Their treatment is extremely helpful to us, though perhaps out of date since it leaves out significant progress in certain areas. A more recent and mathematically oriented textbook by (Myerson, 1991) fills in many of these gaps. Two elementary and extremely accessible primers that are useful for applying the ideas of *game theory* to practical applications are (Dresher, 1981) and (Williams, 1966). Another text that we found helpful was (Osborne & Rubinstein, 1994). It demonstrates how game theory has been applied to fields other than economics. In this same context we found (Ordeshook, 1986) and (Shubik, 1991) also illuminating.

A graphic presentation from the standpoint of setting the historical context is the survey of *game theory* (or as he would see it termed, ***interactive decision theory***) during the 20th Century provided by (Aumann, 1989, p. 1). His outline provides a wonderful overview organized into five main time frames:

- "1910-1930
    - Exclusive Form
    - Strategies
    - Strategic Form
    - Randomized Strategies
    - Individual Rationality
    - Zermelo's Theorem
    - The Minimax Theorem
- "1930-1950
    - Cooperative Games
    - Coalitional Form
    - Solution Concepts
    - Domination, the Core and Imputations
    - Stable Sets
    - Transferable Utility
    - Single Play
    - Expected Utility
    - Applications
    - Continuum of Pure Strategies
    - Computational Methods
    - Mathematical Methods
- "1950-1960
    - Strategic Equilibrium
    - Stochastic and other Dynamic Games
    - Repeated Games
    - The Prisoner's Dilemma
    - Nash's Bargaining Problem
    - The Shapely Value
    - Axiomatics
- "1960-1970
    - NTU (Non-Transferable Utility)
    - Coalitional Games and NTU Value
    - Incomplete Information
    - Common Knowledge
    - Bargaining Set, Kernel Nucleolus
    - The Equivalence Principle
    - Many Players
    - Cores of Finite Games and Markets
- "1970-1986
    - Biology
    - Randomization as an Expression of Ignorance
    - Refinements of Strategic Equilibrium
    - Bounded Rationality
    - Distributed Computing
    - Consistency
    - Cost Allocation"

As in any book or article, there will be biases and restrictions based on the focus. Many books and articles on game theory, such as the one above, have a context set by (Von Neumann & Morgenstern, 1944). For our purposes, however, there are additions that should be added that reflect different biases and advances since the article by (Aumann, 1989) was written:

- Greek Philosophy, economics, (Cameron, 2008)
- Chinese Art of War, (Tzu, 1988), also see (Myerson, 1991)
- The Necessity of the Wager, (Pascal, 1670)
- Utility (Bernoulli, 1738)
- Bayes Theorem (Bayes, 1764)

- The Invisible Hand, (Smith, 1776)
- Decision Theory (Condorcet, 1793), from (Hansson, 1994)
- Greatest Good (Bentham, 1829)
- Happiness (Edgeworth, 1881)
- (Zermelo, 1913), see (Myerson, 1991)
- (Borel, 1921), see (Myerson, 1991)
- Statistical Methods, (Fisher, 1925)
- Nash Equilibrium, (Nash, 1951)
- Objective Probability, 1926, see (Ramsey, 1964)
- Systems Dynamics (Forrester, 1961)
- Decision Analysis (Howard, 1964)
- Club of Rome, (Meadows, Meadows, Randers, & Behrens, 1972)
- Maximization, John Maynard Smith, 1982, see (Myerson, 1991)
- Learning Organizations, (Senge, 1990)

We see from the above time lines that the theory of making decisions is part of a philosophical conversation that has been going on in different countries for over a thousand years. We point to the Art of War, (Tzu, 1988) in China, written over 2000 years ago and the Greek philosopher Xenophon, 4 B.C., *cf.* (Cameron, 2008), as examples. The modern form of *Game Theory* starts with (Zermelo, 1913), (Borel, 1921) and (Von Neumann J. , 1928).

Decision making is about making a choice now that will lead to consequences in the future. It is presumed that we make choices to obtain certain desired outcome in that future. As in engineering problems, it is not always possible to frame the desired outcome as a single number or attribute. Nevertheless, for any given set of desired outcomes, there is an advantage to someone who is adept at predicting the consequences of any decision.

We frame the modern debate as one between predictions being made based on probability arguments (Bayes, 1764) or based on process arguments such as (Edgeworth, 1881) starting in the 18th and 19th century. This debate may well extend further back into history. What we find relevant is that both approaches open up intriguing insights and both have serious flaws. The key is to synthesize these ideas into a consistent theory and test that theory against observation.

The issue of predicting the future based on probability versus knowledge of process is not restricted to the field of decision making. It has occurred in other fields where knowledge of the future would be helpful if not critical. A good example is the simple question of predicting the weather, the accuracy of which can have a strong and positive economic impact. For many years, weather was forecast based on probability as provided by past history recorded in the Farmer's Almanac. Modern meteorology adopted a process view by Bjerknes, (Friedman, 1989), who argued that using the underlying processes of fluids and thermodynamics one could vastly improve weather predictions. He used physical or process models, models based on the principle of least action, which are the basis of today's weather predictions. Though statistical evidence and the probability view are still important, the process view changes the paradigm from the Farmer's Almanac.

In the industrial community, there are large organizations that also predict the future based on statistical data and Bayesian probability. For example organizations may predict the number of resources needed to deliver a product as well as the expected delivery date. Based purely on statistical-historical data (probability view), companies will use such data to make financial commitments. However, by contrast, there are organizations, (Senge, 1990), that use a process approach based on System Dynamics (Forrester, 1961) on which to base commitments. It is not surprising that these two camps oppose each other. For example, the inventor of modern statistical methods, (Fisher, 1925), was strongly opposed to what he termed inverse probability (Bayesian probability, Cf. section 7.3).

For decision process theory, we look at *game theory* in a way that is distinct and different from much of the recent literature, which has focused more on this Bayesian approach and is somewhat opposed to, or at least suspicious of, the process camp. Like debates in other areas, one approach is not necessarily wrong and the other right: each brings useful and indeed essential insights to the table. It is clear that the utilitarian school (Bentham, 1829) was not adequate to deal with the complexities of decisions; it lacked the advances in understanding of what constitutes a decision and the precision of language introduced by (Von Neumann & Morgenstern, 1944) that articulated this understanding.

It is our view however that to make further progress, we need to return to the process approach. In the following sections, we pick out topics from the above time lines that help support this viewpoint. We start with one of the cornerstones of modern economic thought, the ***invisible hand*** (Smith, 1776) and explore its underlying assumptions.

7.2 *Code of conduct*

We gain a better appreciation of ***self-interest*** and ***public-interest*** in our *decision process theory* by looking at their historical context. We start with the related concept of the ***invisible hand***, (Smith, 1776). It is a view of a free market that even though individuals pursue their self-interest, the public-interests will be served. It is remarkably similar to the game theory concept that each person's goal is to maximize their gain. The game theory caveat is that each player understands that other players are attempting to do the same, so each player must consider all the worst options and choose from them the best. We have seen with the prisoner's dilemma, the paradox that arises (chapter 5) when the prisoners' pursuit of their own self-interest leads to a less than optimal choice for their public-interest. The only out for economics in general and game theory in particular, is to believe that there are hidden forces at play that enforce the public-interest, namely the *invisible hand*.

7.2.1 Adam Smith—the invisible hand

An essay by (Joyce, 2001) articulates these ideas in the historical context and points to the relevance of contracts and ***codes of conduct*** with the concept of the *invisible hand*:

> *"...every individual necessarily labours to render the annual revenue of the society as great as he can. He generally, indeed, neither intends to promote the public-interest, nor knows how much he is promoting it. By preferring the support of domestic to that of foreign industry, he intends only his own security; and by directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention. Nor is it always the worse for the society that it was no part of it. By pursuing his own interest he frequently promotes that of the society more effectually than when he really intends to promote it. I have never known much good done by those who affected to trade for the public good.*
>
> "In this passage, taken from his 1776 book *An Inquiry into the Nature and Causes of the Wealth of Nations* [(Smith, 1776)] Adam Smith set out the mechanism by which he felt economic society operated. Each individual strives to become wealthy 'intending only his own gain' but to this end he must exchange what he owns or produces with others who sufficiently value what he has to offer; in this way, by division of labour and a free market, public-interest is advanced."

We see Adam Smith's insight as well as his limitations (Joyce, 2001). The idea of becoming wealthy is based on self-interest and assumes that actions such as stealing are forbidden. The possibility of wealth requires that strict laws be in place. Again we quote (Joyce, 2001):

> "Smith was profoundly religious and saw the 'invisible hand' as the mechanism by which a benevolent God administered a universe in which human happiness was maximised. He made it clear in his writings that quite considerable structure was required in society before the invisible hand mechanism could work efficiently. For example, property rights must be strong and there must be widespread adherence to moral norms, such as prohibitions against theft and misrepresentation. Theft was, to Smith, the worst crime of all, even though a poor man stealing from a rich man may increase overall happiness. He even went so far as to say that the purpose of government is to defend the rich from the poor."

In a well ordered society, self-interest would in fact serve the public-interest if the society has in place persistent rules or codes of conduct. These rules in our *decision process theory* correspond to strategies that are *inactive* and *persistent*. The existence of such rules assumes a level of trust among all members of society that nobody will violate that trust. These persistent rules may operate between individuals, families, cities, nations or regions. Even though it is possible to imagine rules that don't promote the public-interest, for the public-interest to be supported there must exist some rules that all generally adhere to.

We get the following concise summary of Smith's assumptions (Joyce, 2001) and her conclusion along with our comments in square brackets of the relevance to our ***decision process theory***:

- "There is a benevolent deity who administers the world in such a way as to maximise human happiness. [We replace the benevolent deity with the physical world in which the *principle of least action* operates at every level of organization.]
- "In order to do this he has created humans with a nature that leads them to act in a certain way. [We replace human nature with the idea of persistency, which leads to the concept of a payoff field that summarizes an individual's approach to decisions.]
- "The world as we know it is pretty much perfect and everyone is about equally happy. In particular, the rich are no happier than the poor. [Our *decision process theory* agrees only in the sense that it makes no distinction between rich or poor in terms of the mechanisms that operate.]
- "Although this means we should all be happy with our lot in life, our nature (which, remember, was created by God for the purpose of maximising happiness) leads us to think that we would be happier if we were wealthier. [We would say that this and the following statement are about dynamics and change. We provide more physical mechanisms to describe these.]
- "This is a good thing, because it leads us to struggle to become wealthier, thus increasing the sum total of human happiness via the mechanisms of exchange and division of labour.

> "It is clear why Smith says that moral norms are necessary for such a system to work – in order for exchange to proceed, contracts must be enforceable, people must have good access to information about the products and services available and the rule of law must hold."

Thus, the invisible hand mechanism works by encouraging entrepreneurs to respond to what consumers want. In this way the entrepreneur becomes wealthy. The entrepreneur assumes however that when delivering goods, he will be paid. The consumer assumes that when he pays for goods, they will be delivered. The consumer assumes further that the entrepreneur is truthful about what his product will do and honest about what the cost of that product will be.

It is certainly possible to imagine people whose goal in life is to be helpful. There are altruistic people with good intentions. There are also people whose goal in life is to look after their own welfare. These are self-interested people. Smith envisions these two types of people (Joyce, 2001):

> "One extremely positive aspect of a market–based economy is that it forces people to think about what other people want. Smith saw this as a large part of what was good about the invisible hand mechanism. He identified two ways to obtain the help and co–operation of other people, upon which we all depend constantly. The first way is to appeal to the benevolence and goodwill of others. To do this a person must often act in a servile and fawning way, which Smith found repulsive and he claimed it generally meets with very limited success. The second way is to appeal instead to other people's self–interest. In one of his most famous quotes:
>
> *"Man has almost constant occasion for the help of his brethren and it is in vain for him to expect it from their benevolence only. He will be more likely to prevail if he can interest their self–love in his favour and show them that it is for their own advantage to do for him what he requires of them. Whoever offers to another a bargain of any kind, proposes to do this. Give me what I want and you shall have this which you want, is the meaning of every such offer; and it is the manner that we obtain from one another the far greater part of those good offices which we stand in need of. It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own interest. We address ourselves, not to their humanity but to their self–love."*

In other words, Smith assumes that most of us will not respond to an altruistic appeal. Rather we will respond when we see some advantage to self. In this view however, Smith ignores his separate view that contracts must be respected. If we dishonor contracts, there would be no basis for the free market. The honoring of contracts requires an appeal to our humanity. We must *simultaneously* pay attention to our self-interest and our public-interest. There must be a balance. This is required by his "considerable structure" necessary to support the invisible hand.

7.2.2 Prisoner's dilemma and the tragedy of the commons

Thus we see the important distinction relevant to our *decision process theory*. There are underlying assumptions made to any exposition of economics or game theory: There is either a person's ***humanity*** or their ***self-love***. What we add to Smith's view is the need to make the invisible hand visible: we must explicitly identify the strategies that relate to the code of conduct. We must separately identify the strategies that relate to our self-interest. Both sets of strategies are always in play.

It is directly relevant to our discussion here that (Joyce, 2001) brings up the connection of the invisible hand and the prisoner's dilemma:

> "Two people, who are suspected of being accomplices in a crime, are held prisoner in separate, non–communicating cells. The police visit each prisoner and tell both that if neither confesses, each will be sentenced to two years in jail.

> However, if exactly one prisoner confesses, implicating each other, the one who confesses will get off scot−free as a reward and the other, who didn't confess, will receive a punitive sentence of five years. If each confesses and implicates the other, both will be sentenced to three years."

This is a variant of our version of the prisoner's dilemma in section 1.4, which we dealt with in greater detail in chapter 5. She ties the prisoner's dilemma to Adam Smith's invisible hand and relates it to specific examples starting with the ***temptation to default***:

> "We can think of the prisoners as being asked to decide whether to keep a contract they have made with each other (remain silent) or to default (confess and betray the other). Similar choices have to be made all the time in economic society. When two people freely agree to exchange goods or services to their mutual benefit, each must decide whether to try to cheat the other by defaulting, or handing over counterfeit goods, or whether to act in good faith and risk the other party defaulting. Obviously, both parties are better off if neither default than if both default − after all, we suppose they willingly contracted with each other − but each would like to get something for nothing and each is afraid the other will feel the same. The result may well be that the parties are unable to carry out the exchange as arranged and both lose out."

This is no longer just the prisoner's dilemma, but a question of ordinary commerce. The broader question is whether in ordinary commerce, the buyer and seller will both act in good faith or whether one or both will default. The *game theory* answer taken from the prisoner's dilemma is that both will default. Paradoxically, with the free market, Adam Smith was arguing in favor of both acting in good faith. They would hold to his strong belief that contracts would always be honored. The prisoners would not confess, yet Smith maintains they also would act always in their self-interest. Hence we have the paradox.

Contracts (Joyce, 2001) are enforced in our society not by an invisible hand but by the courts and other government entities that are brought in when contracts are not honored:

> "Enforcing laws of contract requires cooperation and resources from someone else − in democratic societies, the courts on behalf of the government and the people. But courts and prisons and police cost money and most of the costs fall on people who were not party to the contract in the first place − who are therefore paying for a service that doesn't directly benefit themselves. Such courts fall into the category of "public good" − we are all better off in a society where the rule of law is upheld − but are not created and maintained by any invisible hand mechanism. Courts are set up deliberately to carry out a public good; and, although they may not always work the way they are intended to, there is nothing unintended about their use to enforce contracts."

In addition to the ***temptation to default***, (Joyce, 2001) suggests another example of the prisoner's dilemma, ***subsidy-seeking***. She argues that in democratic societies, there can be special interest groups that lobby the government for money for their particular interest:

> "In a democratic society, there is a strong temptation for "special−interest" groups to form and lobby the government to provide tax−payers' money to the group in the form of subsidies. Politicians find the prospect of buying the loyalty of the group attractive and the group sees the prospect of getting other people's money for nothing. Clearly, everyone would be better off if no one sought subsidies − by definition, subsidies are only needed for unprofitable activities, that is, activities that other people do not value sufficiently to pay their own money for. However, if other people seek and gain subsidies, anyone who doesn't bother trying to do the same for themselves will end up subsidising others while receiving no subsidies themselves. This fear may force large numbers of people to spend their time lobbying the government for subsidies, rather than simply engaging in more profitable activities − a classic example of the Prisoner's Dilemma and one over which no court has jurisdiction.
>
> "A very similar situation occurs regarding monopolies. Since pretty much every producer is a consumer, it is probably to everybody's benefit overall if no producers attempt to raise prices by monopolising their market; however, attempting to enforce a monopoly can be very attractive to individual producers. Smith rather sardonically observed that
>
> *"People of the same trade seldom meet together even for merriment and diversion, but the conversation ends in a conspiracy against the public or some contrivance to raise prices."*

Looked at in context, our choice of solutions with at least two additional persistent (*inactive*) strategies for the prisoner's dilemma in chapter 5 is understandable. The *contract* for each prisoner is to not confess, so that the total number of *active* strategies is reduced from four to two. We in fact chose one additional strategy to be inactive (the total effort equal to the sum of all strategies). The only active strategy left in this problem in which the players can exercise their self-interest is the relative effort. Of course if we consider problems with more strategic choices available to each player, we get a more interesting result after we allow for the contract or code of conduct restrictions.

We thus learn a great deal from (Joyce, 2001) about the invisible hand and its relationship to the prisoner's dilemma. It agrees with an earlier observation (Rapoport, 1989, p. 199) that the prisoner's dilemma is a version of the ***tragedy of the commons***. If a farmer allows just one more cow to graze on the commons than what has been agreed to, he benefits. However if every farmer were to do this, all would lose because at that point the land would be over grazed. The prisoner's dilemma scenario is not at all uncommon, but one example of the tragedy of the commons. There are many examples (Joyce, 2001), to which we add a few more.

7.2.3 Tragedy of the commons—the blame game

The first example is what is termed the ***blame game***. A team of two people engage in an activity and they collectively fail. For example a team sport such as doubles tennis. The most productive outcome for both is to assess what went wrong and fix it before the next game. However there is a positive, albeit selfish and unproductive outcome for one if he can pin the blame on the other player. Of course this is symmetric and the other player knows that he too would personally benefit if he could pin the blame on the other. If they both end up claiming the other is at fault however, they both come off poorly, though not as poorly as if they have the blame pinned on themselves with no retort. They would be best served with a coach that gives the team focus on winning their sport. A good coach would enforce a ***code of conduct***. The coach's efforts are positive and constructive even though the coach may have no direct role in the decisions made on the court during the game.

If the players adhere to a code of conduct that involves not playing the *blame game*, then they will be better off in their endeavors playing tennis. We emphasize that the *blame game* is not directly associated with the game of tennis on the court. We think this is a general rule that a code of conduct, such as adherence to contracts, is not necessarily related to the business of transactions. So it is not directly associated with the accumulation of wealth in business, but indirectly it is required. The adherence to contracts (Smith, 1776) is required for the accumulation of wealth.

7.2.4 Tragedy of the commons—the freeway game

A second example is the ***freeway game***. We think it obvious that in general, people would get to their destination faster and safer if they adhered to certain courtesies while driving. However, in a world where all adhere, there is an advantage to one individual who does not. This leads to the behavior that most don't adhere to such courtesies and we have another example of the tragedy of the commons.

7.2.5 Tragedy of the commons—the political commentary game

A third example is the ***political commentary game***. Two people can engage in a conversation that is constructive, that involves active listening and leads to new insights. However this again assumes a basic code of conduct. Each person can abuse the listening of the conversation by directing it towards their own self-interest. They play the "it is all about me" conversation game. They achieve a gain over all others assuming that the others acquiesce and listen. Each person however "sees" the same possibility and the outcome can easily be that all engage in the game of the ***me-conversation***. Television talk shows purporting to provide political commentary provide an excellent illustration of this.

7.2.6 Tragedy of the commons—standard of behavior

We see that what is missing from the various examples of the tragedy of the commons is an explicit inclusion of the operative ***code of conduct***. In contrast, in our *decision process theory*, we explicitly identify the code of conduct with persistent inactive strategies and so the code of conduct is an attribute of the solution. Our notion of the code of conduct is very close to the ***standard of behavior*** (Von Neumann & Morgenstern, 1944, p. 265). In their *theory of games*, they suggest that players might form coalitions that are outside the game and make payments or imputations to each other. They suggested however, a specific principle of stability for this standard of behavior that has not received general acceptance. Our notion of persistence is general and we think may prove more useful. It incorporates many of their notions, which align with our discussion here on code of conduct.

7.2.7 Self-interest versus public-interest

Using the word self-interest, we make an assumption about what we mean by an individual or agent in the decision process. The example of the blame game suggests that agents may not always accept responsibility for their actions. But, even taking this stance of not being responsible is itself a choice they make and are accountable for. We suggest that the decision making process always requires an accountable agent, who benefits or suffers from the consequence of his decision. In time of war that consequence could be fatal. The insight of the *Art of War* (Tzu, 1988) is that all decisions are of this nature. Decisions are made on the ***killing field***, not only in time of war. In other words, decisions really matter. Since to be accountable is itself a strategy, the most elementary code of conduct is taking ownership of one's actions. This ownership we call ***self-interest.*** It defines a specific type of person, agent or entity (such as a corporation) who makes decisions. In the market place, we clearly distinguish owners as those who buy or those who sell. Those who do neither are observers. Therefore we identify ***active agents*** as those that make decisions. They ***control*** a subset of active or inactive strategies and act in their ***self-interest*** in the sense they are ***accountable***.

This does not mean that they necessarily seek to maximize their self-interest for any given decision. They may make strategic choices that include whether or not to adopt or adhere to additional codes of conduct. Real world examples of additional codes of conduct abound; some corporations issue along with their mission statement a code of conduct to their employees about how they do business. For example they may explicitly state they don't engage in bribery, though in some overseas locations such practices might be common.

In a more general sense, a code of conduct is a statement of control that encompasses self and public-interest. A code of conduct presumes that each active agent, by convention, controls certain choices. The most fundamental aspect of control is control of ***self***. In our *decision process theory*, we attribute self to a persistent attribute (chapter 3) or *inactive strategy* that might be viewed as a circular symmetry, thus exhibiting the inactive nature of the strategy. All points are equal; effects depend only on the (conserved) speed and direction around the circle. The persistency defines what we mean by self, agent, individual or player. Thus self is the fundamental control, the fundamental code of conduct. We diverge from game theory by realizing that dynamic behavior does not require nor does experience suggest that self is the only control operating when making decisions. We make decisions on the basis of our ***public-interest*** as well as our *self-interest*.

We identify groups, organizations, societies and nations as having a code of conduct when every member of that collection treats a set of strategies as being inactive. These strategies form and define an extended code of conduct. This type of persistency we identify as ***inter-dependence*** and note that it clearly serves the ***public-interest***. A special case of inter-dependence is ***altruism***. It is our view that any individual is free to act based on any combination without bias of public or self-interests. In other words, we argue that the wealth of nations is the result of an enlightened self-interest, in which all participants agree to a common set of policies. Further, we argue that any set of policies can be chosen, not all of which yield an outcome that would be considered individually positive. If all players adhere to a code of conduct, then that code of conduct is an attribute of the solution in our *decision process theory*. It operates like symmetry in physics. The codes of conduct, as stated above reflect the inactive strategies that exist in addition to the self-interest strategies. Such symmetries are as stable as individual symmetries.

As in other engineering problems, solutions can't be simply ordered from best to worst. Choice must be made based on many criteria that include cost, benefit and risk. Good governance depends on the substance of the policies and their acceptance of the participants over time. What engineering can provide is a description of the dynamic behavior of the structures and an analysis of whether the structures will stand up over time.

7.2.8 Strategic control

In general we think of a player or agent as one who controls some subset of the strategies. Because of the code of conduct, we could identify each persistent inactive strategy with an agent. However these agents control no active strategy. In this sense these agents act like the ***dummy*** players of (Von Neumann & Morgenstern, 1944, p. 340). We call such players ***observers***. They observe in the sense of an umpire or referee. They are responsible for a subset of the inactive strategies and so care about the outcomes (section 7.6). It may in fact not be inappropriate to think of real people acting as agent for the collection of strategies we call the *code of conduct*. We think of governments, courts, standards bodies, boards of directors, etc. that create policy but not action. Thus, the player with no active strategy assigns values to outcomes but makes no choices. This person is clearly ***dependent***. It is perhaps amazing that the theory provides a mechanism for such a dependent to have influence.

7.2.9 Standing in line game

As a simple example of the interplay between active and inactive strategies and their relation to a code of conduct, we consider the simple act of standing in line. In some countries such as the United States and England, standing in line is based on an unofficial code of conduct. Three days a week I stand in line waiting for a train. Typically I am the first person there and so get a chance to observe what happens. I stand as close as possible to what would be considered the beginning of the line, which is near the exit door to the train. On my left is a wall and to my right is an open hall. The first person to come after me has a choice to stand by my side staking out an equal claim to be first (an active strategy), standing to my right but behind me or standing directly behind me (admitting that being in the queue is an inactive strategy). Very rarely if ever will someone stand beside me. The inclination to form a queue is very strong. In fact most of the time people will line up behind me, demonstrating this inclination. Sometimes however, a person in fact lines up behind but to the side. There is a competitive issue here. If I hold my place, the next person may well line up behind number two, starting a second line. I have seen this happen with two lines forming. Clearly the hope of the number two person is that this second queue will become the only queue and provide impetus for that person being first. A counter to this behavior is for me to move away from the wall midway between him and the wall. In this case, I observe that the next person tends to see a single queue and lines up appropriately, either behind me or along the wall. Once a sufficient number of people are in line, the whole line including the number two person ultimately gravitates back towards the wall. In this example we see both competitive forces at work as well as cooperative forces. We see the forces that hold the inactive symmetry in place.

The enforcement of the queue is part of cooperative forces (section 7.6). Anyone that attempts to jump the line will be glared at by those in the queue. This ***glaring*** becomes more effective the longer the line. When glaring doesn't have the desired effect, I have observed that people will loudly mumble about the lack of consideration of the person jumping the queue. The next level of enforcement is some people will pointedly tell the line jumper that there is in fact a line and indicate where the end of the line is. These stratagems seem to work almost always. I have never seen the situation escalate to violence, though I suggest that is possible. At this point, the inactive nature of the code of conduct is destroyed and the solution reflects queuing as an active strategic process.

7.2.10 Zero sum strategy games

Our particular solution (chapter 5) to the prisoner's dilemma can be understood now in this new light. For any general two person game, we identify the sum of active strategies of the first prisoner as r_1 and the second prisoner as r_2. We further identify the difference of active strategies of the first player as s_1 and the second prisoner as s_2. The code of conduct is that each prisoner agrees to treat his strategy difference s_k as inactive. Although each prisoner controls his own difference, like the queuing example, there are various levels of enforcement, starting with glaring, that hold these strategies inactive. Furthermore, we identify an additional code of conduct by taking the sum of the two active summed strategies $r_1 + r_2$ to be inactive. This strategy is controlled jointly by the two prisoners. Depending on the specifics, we may also envision all three inactive strategies as being controlled jointly. The sole active strategy $r_1 - r_2$ that remains reflects the *relative player effort* of the two prisoners. This introduces an effective agent reflecting joint control, which is accountable for the relative engagement choice. We think a general consequence is that there will typically be some subset of active strategies that are no longer under individual control.

We thus obtain a solution that has a code of conduct as a consequence of our choice of inactive strategies. We allow each prisoner to make their own choice as long as they adhere to the code of conduct. We obtain as a consequence of them acting in accordance of a code of conduct, the possibility that the solution behaviors can in fact encourage public as well as self-interest.

7.2.11 Accountability and number of agents

It is worth noting that when there is a code of conduct, the resultant game involves effectively more players. The specific prisoner's dilemma solution (chapter 5) involves at least three players; in this numerical example there are five players. Identifying the players in terms of the inactive strategies provides a refinement of what we mean by putting a game into normal form. We need to identify all of the players and further identify the strategies for which they are ***accountable.*** Players that are accountable for *active strategies* are the ***active agents***; those that are accountable only for *inactive strategies* are the ***observer-agents***. ***Even*** though observers are not accountable for active strategic choices, they are accountable for the code of conduct (inactive strategies) and influence actions by other mechanisms such as glaring *etc.,* which we discussed with regard to queues.

The idea of *code of conduct*, make the metaphor of decisions and recreational games stronger. The objection might be that in real life, particularly life and death situations, there are no rules. For example in war, one might argue that the only rule is to win at whatever cost. The counter would be the Nuremburg trials and Geneva Convention. There are rules about prisoners of war, about treating the wounded by the enemy, *etc.*

In summary, we see that not only in our *decision process theory*, but in the *theory of games*, there is a presumed code of conduct that all players adhere to. Without invoking the code of conduct, we may be led into paradoxes such as the prisoner's dilemma and the tragedy of the commons. In our *decision process theory*, we introduce in a natural way the concept of the code of conduct as the manifestation of persistency of inactive strategies (chapter 3). We view self-interest as the persistence associated with an active agent who controls a subset of the strategies. We view public-interest as the persistence associated with solutions in which active strategies are collectively chosen as inactive.

7.3 ***Dealing with uncertainty***

The foundational aspects of *game theory* are associated (Aumann, 1989) with the period 1910-1930. Games as used today are described in two forms: extensive and normal. The ***extensive form*** provides the common sense description of recreational games and realistic decision processes. For example, we understand chess as a recreational game, which is played by two people who take turns making their ***moves*** according to agreed rules. The game ends after few moves or many depending on the relative skill of the players. At the end of the game each player receives (or pays) a payoff. One contribution of (Von Neumann & Morgenstern, 1944) is their framing of general transactions in the economic world as being mathematically the same as an extensive recreational game. A necessary assumption is that in the economic world, there is a utility function that allows us to assign a payoff to the outcome, just as in recreational games. In our foundational game theory, section 1.1, we implicitly adopt this view for decision processes. It is not an explicit adoption because it is difficult though not impossible to frame the theory in the extensive form in such a way that it covers all conceivable games.

It is more convenient to frame the strategic content of the theory in the ***intensive form*** (Von Neumann & Morgenstern, 1944), also called the ***strategic form*** (Aumann, 1989)**.** Using chess again as an example, to most effectively compete, each player could lay out every conceivable scenario on a decision tree, each ***pure strategy***, based on possible moves the two players might make over the duration of the game. For chess the combinatorial possibilities are staggering but finite. Each player would then characterize their strategic choices quite simply: they each choose one of their pure strategies. The game in this *intensive* *form* consists of a ***play***, which is a single choice by each player and the payoff is based on the utility or outcome assigned to each player. Obviously, each play may consist of a large number of moves. The advantage of this view is that we can treat every game in the same way, distinguishing games only by the number of players and the number of pure strategies associated to each player. The intensive form captures all of the essential strategic details of the game or decision process. Two games with the same intensive form should behave equivalently. In our foundational game theory, section 1.1, we have adopted explicitly the view that decision processes can always be described in this way. So far we align with the work from this early period 1910-1930.

In this same early time period, insight was obtained on different types of uncertainty, an insight facilitated by a focus on the intensive form of games. Uncertainty enters game theory in a variety of ways. Already for recreational games, such as card games, there is a well-known element of chance separate from any uncertainty the players may have in making their pure strategy choice. This type of uncertainty can be absorbed into the definition of the pure strategies. We consider as part of the moves, each possible random choice and redefine the pure strategies accordingly. Thus from a theoretical view, there are no new issues on how to treat such games.

A second uncertainty requires a fundamental change to a theory that deals only with pure strategies. It was observed (Von Neumann & Morgenstern, 1944) that the outcome of certain games (*e.g.* exercise 2) is determined by an optimal choice of pure strategies whereas the outcome for others (*e.g.* exercises 3-8) is not. When the outcome is determined, every player can analyze each of his pure strategies based on whatever the other players might do and identify the worst outcomes, which corresponds to the minimum payoffs for each of his pure strategies. The pure strategy that has the largest payoff then would be the best choice from this defensive stance, called the max-min solution. In the chess game example, the max-min solution for each player has a payoff. If the two payoffs are the same, the game is ***strictly determined***. This is the max-min theorem and holds for all strictly determined games. For strictly determined games, the outcome or payoff is the same for every player and each player plays one of their pure strategies. Furthermore, there is no other pure strategy that will give a better outcome irrespective of what the other players might do. In general, games are not strictly determined, so these games need to be dealt with.

For games that are not strictly determined however, the max-min solution though defensive, may not be optimal. Though the intensive form exposes only the strategic properties of the game, it fails to take into account another important aspect of all games and decisions. We know most about games if we play them repetitively. As in scientific inquiry, we have a hard time creating a theory for phenomena that occur once. For strictly determined games, it makes little difference if a game is played once or multiple times. The max-min solution is still the optimal solution.

For any repetitive game that is not strictly determined, any player that consistently plays a single strategy can have their strategy discovered by the other players. Other players may cease acting defensively in hopes of improving their outcomes. Once a player discovers that their defensive strategy choice starts leading to lower payoffs, they will be encouraged to change to some other pure strategy. This in turn will lead the other players to modify their choices. We discuss in more detail below whether a player makes a choice of pure strategies based on a prediction of what other players might do, or makes their choice based on past history of what is no longer a good idea. Either way, each player sees an advantage to diversify their portfolio of choices and choose plays according to some frequency distribution of pure strategies, called a ***mixed strategy***. For players that pick mixed strategies, it can be proved that there is a max-min theorem for two person games (Von Neumann & Morgenstern, 1944). The max-min theorem is replaced for multiple players with an equally effective and equivalent theorem (Nash, 1951). Nash equilibrium is a mixed strategy choice for each player that is optimal in the sense that there is no other choice available to any player that would be superior. To the extent that every game has Nash equilibrium, the *stationary* behavior of games is established. The underlying meaning of mixed strategies however is still open. We address that next.

We share with (Von Neumann & Morgenstern, 1944, p. 19) the view of mixed strategies as a frequency distribution:

> "Probability has often been visualized as a subjective concept more or less in the nature of estimation. Since we propose to use it in constructing an individual, numerical estimation of utility, the above view of probability would not serve our purpose. The simplest procedure is therefore to insist upon the alternative, perfectly well founded interpretation of probability as frequency in long runs. This gives directly the necessary foothold."

We thus think of decisions being made based on knowledge of past events, including past frequency distributions, as opposed to an *estimate* of future behavior. It is our view that these frequency distributions change in time according to deterministic equations based on past behavior. We don't hold that such frequency distributions predict the future using the mathematics of probability theory. In other words, our deterministic equations are not equivalent to Bayesian probability theory (Bayes, 1764) in its various forms.

We make this point because the direction of game theory subsequent to (Von Neumann & Morgenstern, 1944) has been based on probability arguments as the mechanism for future predictions. It is of course tempting to predict the future using a minimum of assumptions about what is known. This is especially true when we know little about the underlying processes. It provides a strong reason for using probability arguments. Decisions are about things we care about. If we can predict the future then our payoffs will be better and potentially better. The question is whether we use probability or an understanding of decision processes to make that prediction. Either way we may use statistical arguments.

The *estimation process* makes a prediction based directly on the statistical evidence. We have referred to this as the Farmer's Almanac approach. In this case, we require little knowledge of the details of past weather phenomena. In contrast, fluid mechanics models make predictions based on the assumption of continuity of the physical process and the observation that these processes obey laws. We need however vast quantities of data about the past in order to utilize such models. In this case, statistical evidence is used along with the ***null hypothesis*** (Fisher, 1925) to look for evidence against such laws. In other words it is not enough to postulate a theory; we have to demonstrate that the theory does not fail. When we identify failures we must modify the theory to correct the fault. The difference of these two approaches was discussed during the formative stages of the development of statistics (Fisher, 1925). We say more about that here.

Much of current game theory relies on Bayesian (Bayes, 1764) or inverse probability as the means to predict the future. The view is that if we observe the occurrence of an event repeating itself in the same way many times, it will be more likely to do so in the future. If the occurrence is a set of strategic behaviors for a game, we may even adopt that behavior as the equilibrium behavior. However, we adopt the opposite view (Fisher, 1925, p. 9):

> "… For many years, extending over a century and a half, attempts were made to extend the domain of the idea of probability to the deduction of inferences respecting populations from assumptions (or observations) respecting samples. Such inferences are usually distinguished under the heading of **Inverse Probability** and have at times gained wide acceptance. This is not the place to enter into the subtleties of a prolonged controversy; it will be sufficient in this general outline of the scope of Statistical Science to reaffirm my personal conviction, which I have sustained elsewhere, that the theory of inverse probability is founded upon an error and must be wholly rejected. Inferences respecting populations, from which known samples have been drawn, cannot by this method be expressed in terms of probability, save in those cases in which there is an observational basis for making exact probability statements in advance about the population in question."

In the context of making decisions, we thus argue against a Bayesian approach. We hold to the notion of frequency (as opposed to lotteries or probabilities) for mixed strategies and the use of statistics as a necessary ingredient of the scientific method, but not a predictor for future behavior.

In our *decision process theory,* whether a person believes they are predicting the future or acting on information from the past will be treated in the same way. We assume that however one arrives at a frequency distribution for mixed strategies, knowledge of that distribution as a function of position and past times provides a scientific basis for future predictions using a calculus based theoretical framework, the *decision process theory*. Our difference from the early *game theory* work is not foundational, yet our differences are foundational with later work. We differ because we believe the focus for a dynamic description is on the repetition of the game, which gives the observational data for the frequency distributions of mixed strategies.

This uncertainty about mixed strategies is closely tied to the conversation about stable solutions. We have characterized this uncertainty as two schools of thought: one that frames probability theory based on set theory, combinatorial methods and Bayesian probability to predict the future; the second, counterpoised to this, frames decision processes based on underlying processes and utilizes differential equations and mathematical tools from the physical sciences. There is evidence to support belief in the former approach (Von Neumann & Morgenstern, 1944, p. 6):

> "The importance of the social phenomena, the wealth and multiplicity of their manifestations and the complexity of their structure, are at least equal to those of physics. It is therefore to be expected—or feared—that mathematical discoveries of a stature comparable to that of calculus will be needed in order to produce decisive success in this field. (Incidentally, it is in this spirit that our present efforts must be discounted.) *A fortiori* it is unlikely that a mere repetition of the tricks which served us so well in physics will do for the social phenomena too. The probability is very slim indeed, since it will be shown that we encounter in our discussions some mathematical problems which are quite different from those which occur in physical science.
>
> "These observations should be remembered in connection with the current overemphasis on the use of calculus, differential equations, etc., as the main tools of mathematical economics."

In support of their view, we see value in the axiomatic tools that have been applied that have provided much needed analysis and clarity of distinctions about what constitutes decisions in general and economic decisions in particular.

In contrast, we are not in agreement that "tools of the physical sciences are inappropriate". We believe a number of advances have been made since that statement was written, as well as a few setbacks. The particular complexity, namely the existence of a stable ***standard of behavior***, suggested by (Von Neumann & Morgenstern, 1944) does not always exist (Aumann, 1989, p. 13):

> "Von Neumann and Morgenstern were thus led to the following definition: A *set* K of imputations is called stable if it is the *set* of all imputations not dominated by any element K :
>
> "This definition guarantees neither existence nor uniqueness. On the face of it, a game may have many stable sets, or it may have none. Most games do, in fact, have many stable sets, but the problem of existence was open for many years. It was solved by Lucas (1969), who constructed a ten-person TU coalitional game without any stable set. Later, Lucas and Rabie (1982) constructed a fourteen-person coalitional game without any stable set and with an empty core to boot."

Moreover, there is new understanding that shows that the differential equations of physics allow for vastly more complexity (Thom, 1975) than was previously believed, complexity of the type that might in fact come out of the set theory and combinatorics view. See for example (Chandrasekhar, 1961), which has particular relevance given the similarity of our *decision process theory* to the theory of relativistic charged fluids.

In the connection to relativistic charged fluids, we note similarity to solutions there and complexity demonstrated numerically in our solutions to the prisoner's dilemma such as Figure 5-37. One of the striking predictions of *game theory* that rested entirely on the algebraic and abstract formulation was the standard of behavior. In section 7.2, we provided a view of codes of conduct, which we feel is an improved definition of that concept. This improved definition satisfies both the set theoretic basis in symmetry and the process basis consistent with a calculus based approach.

The concept of standards of behavior or codes of conduct, along with the ability to frame any decision process in intensive form provides the starting point in our *decision process theory*. We require primarily knowledge of the earliest period of game theory, 1910-1930. Fortunately, many of the elementary text books on game theory address the question of how to frame games in the intensive form. We benefitted greatly from one such, (Williams, 1966) and suggest the student work through their examples. We provide exercises at the end of the chapter, some of which are from this reference.

In this section we have raised the notion of uncertainty and discussed our view of probability and statistics. This viewpoint changes how we view utility, which we turn to in the next section.

7.4 ***Utility Theory***

Utility is the value or worth we give to things we buy or sell. We assign utility to actions we take towards others and actions others take towards us. In commerce we believe mechanisms are in place to value goods and services making it possible for a market to exist. More generally, all decisions require a notion of value associated with the outcomes of that decision. To discuss value or utility, there is no loss in generality to discuss the decision process in normal form. In normal form, each agent makes a single pure or mixed strategy choice. The resultant outcome is that each agent will give or receive something of value dependent upon the collective choices made (and dependent on when that choice was made). From a theoretical standpoint therefore, it is critical that this concept of value or utility be made precise. The basis of a quantitative theory of decision processes rests on having a quantitative theory for utility. This has been done through an axiomatic approach, which provides the necessary mathematical infrastructure for a precise and usable definition.

7.4.1 Preferences

In the axiomatic approach of game theory, utility is discussed in the context of how decisions deal with certainty, risk and uncertainty (Luce & Raiffa, 1957). A simple decision expresses a ***preference*** or choice between alternatives. If we choose between two distinct alternatives A and B, then we obtain our choice with ***certainty***. If we choose between an outcome A and some mixture of B and C, then we obtain an outcome with ***risk***. If we pick the mixture we know only the frequency with which we might get B or C but we don't know which. If there are other players or factors that affect our decision, then we obtain an outcome with ***uncertainty***. This verbiage suggests that our preference is a prediction of what we will obtain in the future based on our choice. It includes an ordering based on what will probably happen. These authors (Luce & Raiffa, 1957) go on to discuss utility in terms of lotteries and prizes, a context that is suited to the modern game theory approach (Gardenfors & Sahlin, 1988) in which equilibrium behavior follows from the ***rational behavior*** of the participants. This context favors the idea that probabilities are like lotteries and are *estimates* for future behavior and potential gain or prizes. The context makes possible discussing future behaviors in terms of certainty, risk and uncertainty.

In this section we wish to recontextualize utility so that it conforms to our general *decision process theory*. In our view, we don't envision predicting the future (section 7.3) based on probability. We frame our view in an historical context. Though an early treatment of utility goes as far back as (Bernoulli, 1738), the treatment that best meets our needs starts with (Von Neumann & Morgenstern, 1944), which is based on frequency of occurrence rather than probability.

Their idea is that it is self-evident (in the sense of creating axioms) that between two alternatives, A and B, from past behaviors a decision maker can choose the one that has more utility. This knowledge is not enough however to provide a basis for utility in game theory. A decision maker needs to know more. Between the alternative A and the alternative of some combination of B and C, again by past behaviors a decision maker can choose the alternative that has more utility. When the concept of mixture is made precise, a numerical utility function can be defined that expresses a utility for any mixture of choices. They adopt the most conservative approach, which is that the mixture is based on the frequency of occurrences of choices B and C in the past. This approach is consistent with the usual application of the scientific method (Fisher, 1925), as it does not invoke Bayesian or inverse probability (section 7.3).

The issue of how we predict the future becomes more complex when we deal with more complex decisions than picking preferences. Now we have issues of uncertainty in our outcomes based on decisions made by others or factors outside of our control. We hold to the view that despite these complexities, a process view is possible in which the future flows from past behaviors of all the agents involved in the decision process. We may be able to calculate that flow but only if we make assumptions about those underlying processes. Hence, we make no use of probability as *estimation*.

We do make use of probability or statistics in the usual way it is employed with the scientific method to discredit assumptions that appear extremely unlikely. Thus in our dynamic theory we base our assumptions of what is known about decisions in the present and past and use the *decision process theory* to determine future behaviors. We use the scientific method to modify our theory based on observations and measurements.

We also use probability and statistics to measure. For example when we measure the distance between two points, we do the measurement several times and average the values obtained. There are differences each time based on the uncertainties of the measurement process that are unrelated to the concept of length. In decision processes, there are uncertainties in measuring the utilities that are unrelated to the presumed concept of utility between A and a given combination of B and C.

Thus our *decision process theory* allows us to use past data on frequencies to make choices. Based on this approach, we expect the axiomatic proof of utility (Von Neumann & Morgenstern, 1944) to follow unchanged. As a consequence, we should be able to compare frequency distributions of each decision maker and allocate a numerical utility, the payoff, to each of them. In order to assure that we can carry out this program, we envision that sufficient repetitions of the decision process can be observed in order to arrive at reasonably accurate measures. We thus specify what we mean by a point in time: it is an interval that is large enough to carry out the above program, though not so long as to open the discussion of what behaviors might be *stationary* (*Cf.* section 2.7).

We believe there are numerous examples of this process used today in the commercial world, examples that assign time-dependent numerical utilities and payoffs to processes that are inherently human and subjective. Consider as an example the production of code by software developers. The production of code results from a collection of decisions. If the decisions were correct, the code works; otherwise the code has parts that don't work and has defects that will be identified in the discovery or testing stage. The percentage of code that works versus code that doesn't work represents a frequency distribution that is knowable in principle at the time the code is produced but in practice is not known until later. One can nevertheless model the production of code as a quantity of code with a certain defect fraction. The defect fraction is not an equilibrium statement. It depends on the skill and experience of the software developers. Over time and based on feedback during the discovery stage, the number of defects will go down. This is an example of a strategic fraction changing in time as well as a utility that increases over time as the production quality increases. To improve profit and customer satisfaction, the software development company provides feedback loops to decrease defects, increase skill levels and thus lower cost and improve quality. A static view using frequencies as estimates of future behavior would miss this insight.

We assert that it makes sense to consider the cardinal as well as ordinal properties of utility based on comparing choices that involve combinations with frequencies (Von Neumann & Morgenstern, 1944, p. 617). For choices that involve certainty, only ordinal utility is needed. The consideration of mixed strategies however brings us back to a cardinal view (up to linear transformations). There appears however, a belief that remains that the cardinal view is too restrictive and only the ordinal view is needed. It is framed as the view that only the set theoretic view is needed because the calculus view lacks the capability to produce complex effects (Luce & Raiffa, 1957, p. 18):

> "The problem is to find an act satisfying … [the linear programming problem]. It is clearly a decision-making problem under certainty; however it cannot be handled by the traditional methods of the calculus. What is known as the theory of convex bodies has proved crucial."

We have demonstrated in our *decision process theory* that dynamic stable points of games will also satisfy the equilibrium condition and hence the theorems of convex bodies.

We now know that complex algebraic results are not excluded as attributes of solutions to differential equations. Our adoption of a differential geometry approach allows the possibility of local behaviors in which each individual player can adopt their own view of the payoffs and utilities without making it impossible to compare those utilities and without assuming their utility measures are the same.

We thus see no reason to exclude calculus as incompatible with many of the algebraic results that have been obtained, including those on utility. We see no reason to expect that calculus is any less rich in allowing complex structure than algebra. The modern view of calculus is that it is a rich combination of topology, group theory and algebraic structure. This rich structure and its possibilities are amply supported in the literature. For example it is viewed as a gauge theory by (Hawking & Ellis, 1973, p. 50) and viewed in the mathematical world as a fibre bundle topological structure with a connection (Eilenberg & Steenrod, 1952). For introductory lectures on the subject of both these physical and mathematical views, see (Thomas G. H., 1980).

7.4.2 Local utility

We restate two caveats on utility that relate to our *decision process theory* introduction in chapter 1, which echo and generalize a similar discussion in (Luce & Raiffa, 1957, p. 33). First, the utility function is determined up to a linear transformation and is specific to each decision maker. Second, we make the further restriction that this utility function is local (the gauge theory property), so it is determined at each point in time and space. We see utilities as reflecting the underlying measure of distance between strategic points in space and time. This underlying distance measurement or metric may vary and evolve over time according to dynamic rules, rules that we have specified in our *decision process theory*. In our theoretical framework, the metric determines the payoff tensor, *cf.* section 1.9, Eq. (1.73). We can relate the payoff tensor to the utility of (Von Neumann & Morgenstern, 1944) using the following argument that helps articulate their general framework.

To set the stage, we assume as an example that there is some utility measure in business transactions. Suppose we have invested in three companies. Our experience is that our investment in company A yielded \$25K, company B yielded \$30K and company C yielded \$10K. Based on this experience we ***prefer*** A to C and B to A. Because we are using a numerical measure, we see that we also prefer A to a 50% investment in B plus a 50% investment in C. This is because a ½ interest in each of B and C would have yielded \$20K, less than the return from A.

The insight (Von Neumann & Morgenstern, 1944) is that this argument can be turned around. First if we prefer A to B and assign a numerical utility $u(A)$ to A and $u(B)$ to B, then because of our preference, the numerical utility of A should be larger than the numerical utility of B: $u(A)>u(B)$. The standard economic argument concludes only that it is reasonable, therefore, for there to be a utility function u but it is unique only up to transformations that preserve order. The second and crucial step is the assumption that if we prefer A to a ***fractional investment*** f in B and $1-f$ in C, then the numerical utility of A should be larger than the weighted sum of B and C using the fraction f:

$$u(A)>fu(B)+(1-f)u(C) \tag{7.1}$$

Before drawing their conclusion, we note that the word *investment* was used here because of our starting example. We don't however require a concept of money, but a concept that we are willing to split our effort between B and C by some fraction. In the context of our *decision process theory*, the simplest way to envision splitting our effort is by our actions. We thus envision that we prefer putting our effort in A to splitting our effort between B and C based on the fraction f.

If this is the case, then the numerical utilities will be related in the above way, Eq. (7.1). These ideas are made mathematically precise by (Von Neumann & Morgenstern, 1944), who show that under reasonable mathematical assumptions, the utility function is determined up to a ***linear transformation***. In other words if there is any other utility function $\bar{u}$ that also satisfies Eq. (7.1), then there will be numerical constants $\{a,b\}$ that relate the two utility functions for any choice A:

$$\bar{u}(A)=au(A)+b \tag{7.2}$$

They thus arrive at the converse of our starting investment example.

7.4.3 Payoffs

We make connection to our payoff fields as follows. We apply this utility function to decisions in their normal form. Each agent makes a decision by choosing a single pure strategy. We make the assumption that each agent can not only assign a preference to her pure choice versus any set of pure choices of the other players, but can assign a choice based on fractional combinations of choices as defined above. Making the same assumption as before, it is clear that each agent has a utility function with which to compare any fractional choice of her strategies against any possible fractional choices made by each of the other players. This utility function is determined up to a linear transformation. It is clearly determined at some specific time and relative to some specific context of choices that have been made. In other words it is determined locally.

This utility function provides for each agent, the elements of what we have called the payoff tensor. The values will be for each pure strategy of that agent against each pure strategy of each other agent. It is also clear that we obtain a new feature, an ***internal preference*** or ***internal payoff*** for pairs of strategies of the same agent. We see that the payoff tensor captures and extends the essence of the utility argument. In our *decision process theory* we in fact don't claim nor require it to be unique. In the theory there is the concept that at each moment in time as well as at each strategic point, the theory is non-unique up to a set of general linear transformations. We extend the concept of such transformations to make them local. We also insist that the theory be causal in the sense of physical theories, that we don't predict the future but base our calculations on past behaviors that obey laws dictated by a principle of least action. We return to this discussion in section 7.7.

Game theory provides significant insight into payoffs and how agents use them to deal with competition. We take this up in the next section.

7.5 ***Dynamic law of competition***

Game theory provides valuable information on the ***known behaviors,*** *cf.* section 7.9, based on a well developed literature on *stationary* solutions characterized by strategic flows and payoffs. For example in the prisoner's dilemma, chapter 5, we take flows and payoffs to hold along some hypothetical set of strategic values υ_0. We must augment our knowledge from *game theory* to provide a complete specification of the known behaviors. We find that there are additional strains, section 5.5, as well as additional stresses, section 5.6, which must be specified. We use this augmented set of known behaviors, along with the partial differential equations to determine behaviors at other points of time and space.

The *stationary* behaviors provide our starting point. A large component of the *known behaviors* is based on competitive behaviors. For example, the easiest *stationary* behaviors to determine are for two-person non-cooperative games, *i.e.* ***competitive games***. The easiest of these, by far, to deal with are those in which each player has exactly two pure strategies. We use this as an informative and illustrative starting point and then extend our discussion to non-cooperative $M \times N$ two-person games where the two players have M and N strategies respectively. We point out that numerical solutions can always be found for such games using linear programming (Luce & Raiffa, 1957). We conclude by postulating an ***economic equivalence principle*** between the linear programming solutions and our *decision process theory*. As a result we argue that our *decision process theory* provides a law that is the natural extension of *stationary* competitive games to general dynamic decision processes. The ideas of ***dynamic competition***, ***code of conduct*** (section 7.2), and ***dynamic cooperation*** (section 7.6) provide the dual notions of non-cooperative and cooperative games in our theory. We see the decision process as the interplay of these mechanisms.

7.5.1 Game theory limit

The competitive aspect is seen clearly in the two-person zero-sum 2×2 non-cooperative game, which is characterized by the payoff matrix for player 1; the payoff for player 2 is the negative of this, since we consider a zero-sum game. The 2×2 sub-matrix G_{rs} for player 1 contains all the information about this game.

$$\mathbf{G} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \tag{7.3}$$

The two players choose pure or mixed strategies in an attempt to achieve the best outcome and prevent the worst consequences. Two situations can arise: first, there may be one strategy that ***dominates***, *i.e.* one strategy is clearly better than the other. Suppose the numerical values of the payoffs are such that $\{G_{11} \ge G_{21} \quad G_{12} \ge G_{22}\}$. There is no advantage for player 1 to ever choose the second strategy (row 2), since no matter what player 2 does the first row dominates each element of row 2. If it also happens that one column dominates for player 2, then the game is said to be ***strictly determined***. The ***stationary solution*** is for each player to pick their dominant strategy. We see that the definition of *stationary* behavior is the equality:

$$\max_r \min_s G_{rs} = \min_s \max_r G_{rs} \tag{7.4}$$

The most defensive strategy for each player is for them to consider the worst that can happen and then pick the best of these worst cases. It is not always possible however to find such a *stationary* solution since not all games are strictly determined.

For games that are not strictly determined, it is still possible to find a defensive strategy for two-person zero-sum non-cooperative games using mixed strategies (Von Neumann & Morgenstern, 1944):

$$\max_U \min_W \sum_{rs} U^r G_{rs} W^s = \min_W \max_U \sum_{rs} U^r G_{rs} W^s \tag{7.5}$$

Player 1 chooses a normalized mixed strategy $U^r : \sum_r U^r = 1$ and player 2 chooses a normalized mixed strategy $W^s : \sum_s W^s = 1$. In the space of all such normalized mixed strategies, player 1 considers the worst that can happen for each mixed strategy choice that player 2 makes, and then considers the maximum of all of these. Player 2 makes the corresponding defensive choice. The mathematical theorem is that there will always be at least one solution to this problem. The ***stationary behaviors*** for this class of games will be these defensive solutions. The *stationary* behaviors provide a rule or association of strategic choices to the given payoff matrix elements.

For strictly determined games, we can often determine the dominant strategies by inspection. It is in general more complicated to determine the mixed strategy *stationary* behaviors for games that are not strictly determined. There is one case however that is particularly easy: again it is the 2×2 game. We consider such a game in which neither player has a dominant strategy. We compute the following differences for each row and column as follows:

G	$pl21-1$	$pl2-2$	$odds1$
$pl1-1$	G_{11}	G_{12}	$G_{22}-G_{21}$
$pl1-2$	G_{21}	G_{22}	$G_{11}-G_{12}$
$odds2$	$G_{22}-G_{12}$	$G_{11}-G_{21}$	$G_{11}+G_{22}-G_{12}-G_{21}$

(7.6)

Since neither player has a dominant strategy, either the row and column elements labeled odds will all be positive or all negative. If they are all negative, we multiply the elements by minus one. It can be demonstrated that the *stationary* behavior Eq. (7.5) occurs when player 1 picks a mixed strategy based on the odds1 column and player 2 picks the mixed strategy based on the odds2 row. To get the normalized mixed strategy, divide the odds by their sum, given in the odds2-odds1 entry.

7.5.2 Linear programming

The ease with which one can compute the mixed strategies for 2×2 games makes them attractive as examples (*Cf.* the exercises at the end of this chapter), but not representative of the richness present in zero-sum non-cooperative games. By increasing the number of pure strategies for each player from the 2×2 case, the decision process often can be made significantly more realistic. It also becomes much tougher to find the *stationary* solutions by hand or by calculator. In addition to the numerical complexity, there will be combinatorial complexity since there may be dominant strategies as well as mixed strategies. The practical approach to obtaining solutions for such games is ***linear programming*** (Luce & Raiffa, 1957).

We apply *linear programming* to competitive games, namely two-person zero-sum non-cooperative games. A competitive game is described by a sub-matrix G_{rs}, where the first and second indices span the $M\times N$ pure strategies of player 1 and player 2 respectively. We look at the game from the perspective of player 1. He expects to receive a payoff v or greater by playing a normalized mixed strategy $U^r : \sum_r U^r = 1$:

$$\sum_r U^r G_{rs} \geq v \tag{7.7}$$

In game theory, the *stationary* strategies are unchanged by adding a constant payoff to every pair of pure strategies, so in that theory there is no loss in generality to assume that the payoff matrix elements are positive, which implies that the expected payoff to player 1, the best he can hope for, is also positive. In a competitive situation, Player 1 wishes the payoff to be as large as possible. We can divide the inequality Eq. (7.7) by this positive unknown, redefining the mixed strategy $u^r = U^r / v$, which no longer is normalized. The resultant problem is that of finding the minimum of $\sum u^r$ subject to the constraints:

$$\sum_r u^r G_{rs} \geq 1 \tag{7.8}$$

This problem is the *linear programming* problem whose solution is part of standard packages of software such as Mathematica, the one we have used for our calculations.

In like fashion we analyze the game from player 2's perspective. She expects to receive a payoff $-v$ or more, which translates to an expectation that player 1 receives the amount v or less, by playing a normalized mixed strategy $W^s : \sum_s W^s = 1$:

$$\sum_s G_{rs} W^s \leq v \tag{7.9}$$

Again, because the game is competitive, Player 2 wishes this value to be as small as possible. We can divide the inequality by this positive unknown, redefining the mixed strategy $w^s = W^s / v$, which no longer is normalized. The resultant problem is that of finding the maximum of $\sum w^s$ subject to the constraints:

$$\sum_s G_{rs} w^s \leq 1 \tag{7.10}$$

This problem is the *linear programming* problem dual to the previous one. The simultaneous solution of these two linear programming problems provides the max-min *stationary* behaviors Eq. (7.5).

In our *decision process theory*, we have used the fact (section 1.4) that every game can be put into symmetric form (Luce & Raiffa, 1957). We solve the two linear programming problems simultaneously using the symmetric game form. For any game we add a constant to each element so that the symmetric form elements are all positive. We use linear programming on the modified symmetrized game matrix to find the strategies $\{u^r \quad w^s\}$, which for convenience of notation we write as u^a. Given the linear programming solution, we revert to the original symmetrized payoff:

$$\begin{gathered} u^a \geq 0 \\ \sum_a u^a F_{ab} = 0 \end{gathered} \tag{7.11}$$

The linear programming solution of the game is equivalent to finding the null vector of the symmetric payoff F_{ab}. The null vector is what we call the ***payoff direction***, of which there may be one or more. In our *decision process theory*, we expect *stationary* behavior if it is identical to the *payoff direction* of each player.

7.5.3 Decision process theory solutions

Not all games are limited to two players, not all games are non-cooperative and not all games are zero sum. The general treatment of such games nevertheless has much in common with the games described so far. We ascribe a code of conduct to cover persistent cooperation, cooperation that survives all dynamic processes. The code of conduct provides a payoff matrix for each player (including pseudo-players that owe their existence to the code). As argued in section 7.4, we remove the constraint that the utilities of each player are zero-sum or transferable. Still, under rather general conditions we expect that there will be *stationary* behaviors, Nash equilibrium (Nash, 1951), which are determined by the payoff directions. Such behaviors have the attribute that they assume all players behave rationally and assume that with rational behavior, no player can expect to benefit more that what is prescribed to the *stationary* behavior by deviating from that prescription. In many cases we obtain such prescriptions by inspection, which we did for the prisoner's dilemma, chapter 5.

In the context of our *decision process theory*, we see that the various prescriptions for computing (or proving the existence of) the *stationary* behaviors can be summarized as stating that for the collection of all the players' payoff matrices, there is a rational and compelling rule that associates a *stationary* flow. In our *decision process theory*, we specify such a rule, which is a consequence of the field equations, which determines the flow given each of the player's payoffs, Eq. (3.14), a generalization of Ampère's law, Eq. (1.24). In electrical engineering, this is the statement that the incremental sum of the magnetic field around a closed path must equal the current enclosed. If we imagine that for each player, the payoff matrix is roughly constant in a tube surrounding the *stationary* path, then Ampère's law implies that the current that generates the payoff field would spiral around the outside of that tube. If the tube is not very big, then the average motion of the current and the *payoff direction* are the same. We propose that our generalized Ampère's law Eq. (3.14) extends and replaces the max-min rule Eq. (7.5). We propose that it also provide a correspondence to Nash equilibrium for more general games.

7.5.4 Law of competition

We thus propose a general rule that extends the ideas of competition based on *stationary* behaviors. The rule reflects two aspects. First, competitive behaviors are based on a dynamic ***law of competition*** for obtaining the flow vectors from the payoffs, Eq. (3.14):

$$\tfrac{1}{2} g^{bd}\partial_b\left(g^{ac}\gamma_{jk}F^k{}_{cd}\right)=\kappa T_j^{\ a} \tag{7.12}$$

Though this provides the law of competition, it nevertheless depends on the ***cooperation potential,*** γ_{jk}, *Cf.* section 7.6. The interaction is the product of the gradient of the cooperation potential and the payoff matrix *(Cf.* Eq. (3.16)). Second, *stationary* behaviors must reflect, as their name implies, no acceleration of the flow, Eq. (7.15). This ***closed-loop*** behavior provides unity between our *decision process theory* view and the *game theory* view. The ***economic equivalence principle*** is that our view and the max-min rule are the same for two-person zero-sum non-cooperative games.

We support our proposal with the observation that the stress in Eq. (7.12) is the product of the flow and ***player interest flow***, Eq. (2.43), $T_j^{\ a}=(\mu+p)V_jV^a$, which is the conserved current associated with player j. Each player is characterized by a different ***interest flow density*** (*Cf.* producers and consumers, section 7.6) given by the flow component V_j. The *player interest flow* determines whether the flow spirals clockwise or counterclockwise around the (null vector of the) payoff. The orientation of the spiral indicates whether the player is a giver or taker in the decision process.

The simplest case is that *interest flow densities* are all equal and there is a single null vector common to all players. In this case we recover the case above in which the flow of the interest produces a current that generates each of the player payoffs. We see other options however. The *interest flow densities* need not be large at the same points. Players could in fact pursue different strategies that are nevertheless optimal. In other words, the *payoff direction* for one player need not be the same as another. So the dynamic behaviors might in fact be different tubes corresponding to each player that might propagate in isolation, interact or scatter, and then propagate again in isolation as a complicated topological shape. This is *stationary* behavior only if both Eq. (7.12) and (7.15) are satisfied.

Closely related to the dynamic law of competition is therefore the dynamic law of cooperation, which we deal with next.

7.6 *Dynamic law of cooperation*

To initiate our discussion, we follow (Luce & Raiffa, 1957, p. 114), starting with their list of assumptions normally made about cooperative decision processes:

1. "All preplay messages formulated by one player are transmitted without distortion to the other player.

2. “All agreements are binding, and they are enforceable by the rules of the game.
3. “A player’s evaluations of the outcomes of the game are not disturbed by these preplay negotiations.”

The cooperative decision processes are those in which the players make binding decisions that hold throughout the duration of play, what we termed a code of conduct in the previous section. Our starting point is (Luce & Raiffa, 1957, p. 118):

> “The cooperative two-person theory of von Neumann and Morgenstern (1947) singles out the negotiation set as the ‘cooperative solution’ of the game. In words, the players act jointly to discard all jointly dominated payoff pairs and all undominated payoffs which fail to give each of them at least the amount he could be sure of without cooperating. They have argued that the actual selection of an outcome from the multiplicity of points in the negotiation set $\mathcal{N}$ depends on certain psychological aspects of the players which are relevant to the bargaining context. They acknowledge that the actual selection of a point from $\mathcal{N}$ is a most intriguing problem, but they contend that further speculation in this direction is not of a mathematical nature—at least, not with the present mathematical abstraction.”

We anticipate that cooperation requires bargaining (Harsanyi, 1989) and arbitration (Luce & Raiffa, 1957).

7.6.1 Game theory limit

Their specific example helps solidify our understanding of cooperation as described in the theory of games. We consider the set of payoffs for the two players:

$$\mathbf{G}^1 = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}, \quad \mathbf{G}^2 = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \tag{7.13}$$

At the outset, we see that the two payoffs are *not* zero-sum. In section 7.5 we show how to compute the strategies for competitive games. Using those techniques, we compute the most defensive strategy, which is each player assumes no cooperation with the other player. We assume that each player plays a competitive game assuming his own payoff matrix. The ideal strategies for player 1 are then to play his strategies with the odds 2:3. We would say that player 1 is playing against an imaginary opponent, his view of how player 2 would play. This imaginary player 2 would play the odds 2:3. The real player 2 however plays the odds based on his view of the decision process and plays the odds 3:2. His view of the imaginary player 1 is that that player would play the odds 3:2 as well. The real player and imaginary player don’t align. If each player only played this defensive strategy, they would each receive a payoff of $1/5$. We call these strategies the max-min or defensive solutions.

The players might however alternate between the first and second choices in synch with each other, so they would receive the payoffs $(2,1)$ or $(1,2)$. The average payoff $3/2$ to each player for this behavior is much better. To achieve this improved payoff, the players must cooperate with each other. The difficult issue is the nature of the cooperation and how to compute the outcome of that cooperation while still appropriately taking into account the competitive nature that lurks behind each player’s sense of utility (*Cf.* section 7.5). One solution (Von Neumann & Morgenstern, 1944) is to consider the possibilities for the two players viewed in the two-dimensional space of their utilities. Players will cooperate if they can achieve more than what they could do based on the max-min solutions. This defines the sub region of the utility diagram in which cooperation can occur. Cooperation can be persistent, in which case we invoke the code of conduct argument from the previous section. Alternatively, cooperation can be dynamic.

7.6.2 Decision process theory solutions

In this case, we find that cooperation does not replace the competitive nature (Cf. section 7.5) of the decision process but adds additional constraints. We propose as fundamental, the ***law of cooperation***, Eq. (3.21), stating that the ***cooperation potential*** γ^{jk} between player j and player k is determined by the payoffs of the two players and the inertial stress between them:

$$\tfrac{1}{2} g^{ab}\partial_a\left(\gamma^{jl}\partial_b\gamma_{lk}\right) = \tfrac{1}{4} F^j{}_{ab} F_k^{ab} - \kappa T^j{}_k \tag{7.14}$$

For a simple but illustrative model of the stresses, Eq. (2.43), the stress between two distinct players $T^j{}_k = (\mu + p)V^j V_k - p\delta^j_k$, is proportional to the product of the ***player interest flows***. In words, unless the players share a ***common ground***, there is no cooperation. The *common ground* arises in Eq. (7.14) because their ***interest flows*** (charges) overlap, their payoffs overlap or both.

The cooperation potential depends not only on the overlap but on the relative sign of the two *player interest flow*s. This is different from game theory considerations. We frame the sign of the *player interest flows* into two distinct economic categories: ***producers*** and ***consumers***. A more prosaic description for any decision process is ***givers*** and ***takers***, respectively. A market choice would be ***sellers*** and ***buyers***, respectively. Our convention will be to take consumers with the positive sign of *interest flow* and producers with the negative sign. A recreational game is typically between consumers. In this sense our distinction does not arise. Moreover, in a two-person decision process, we expect the payoff matrices for a buyer and seller to subtract. Instead of a zero-sum game for example, we might get a zero-payoff game as a consequence. There will be no payoff forces, only cooperative and inertial forces. We can see these possibilities by looking at the expression for the forces, which we do next.

There will be different types of cooperation depending on the mix of givers and takers in the decision process. The cooperation law leads to the conservation law Eq. (4.54), and with sources q_a, Eq. (2.42):

$$g_{ab}\frac{dV^b}{d\tau} = -\varpi_{abc}V^bV^c + V_kF^k_{ab}V^b - \tfrac{1}{2}V_jV_k\partial_a\gamma^{jk} + q_a \qquad (7.15)$$

This provides the *closed loop* (*Cf.* section 7.8) result that the time rate of change of the flow (the acceleration) depends on inertial (or frame) effects (ϖ_{abc} term), competitive effects (that depends on the payoff) and cooperative effects (that depend on the cooperative potential). We discuss the source contributions in section 7.7.

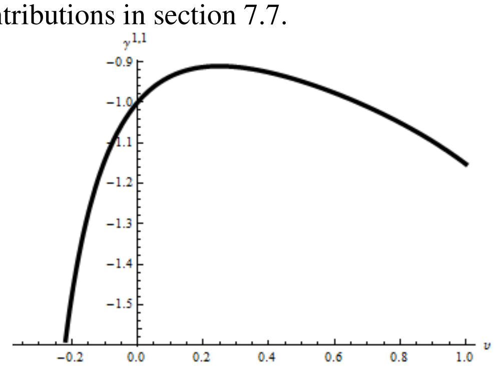

Figure 7-1: Prisoner's dilemma $\gamma^{11}(\upsilon)$

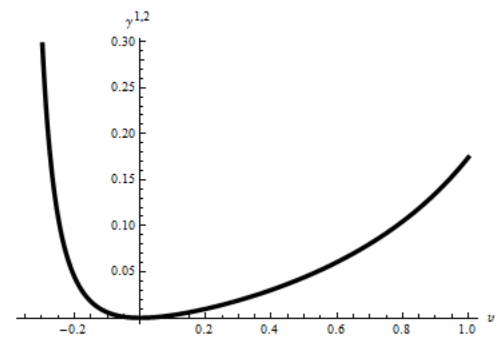

Figure 7-2: Prisoner's dilemma $\gamma^{12}(\upsilon)$

Assuming only cooperative effects, *stationary* behavior occurs when at a maximum or minimum potential of the cooperative potential. Since the product of the *interest flows* and cooperation potential involve the square of each *player's interest flow*, it is reasonable to expect the overall acceleration or force to be a maximum at equilibrium. In that case, moving away from the point would be a restoring (negative) force. This would hold for the autonomous cooperation potential (the self-cooperation effect, Figure 7-1) as well as the cooperation between distinct players. We might further expect that the cooperation potential for distinct players would be a maximum when the *interest flows* have the same sign (both givers and both takers) and a minimum when they have opposite signs, Figure 7-2. Based on the model from chapter 5, we see from the above two figures that our expectations are qualitatively borne out. The model also demonstrates that each player may exhibit both buyer and seller attributes, depending on the strategic value, Figure 5-17. The theory provides a quantitative realization that incorporates all the effects.

7.6.3 Law of cooperation

We further explore the law of cooperation with practical illustrations. I had a long argument with a former colleague some time ago about the distinction between consensus and coalitions. These two ideas often come up in the context of cooperation. He suggested the following distinctions (Laves, 1994):

> "***Consensus***: 'The process of abandoning all beliefs, principles, values and policies in search of something in which no one believes, but to which no one objects'. Margaret Thatcher, 1993.
> "***Coalition***: The art of enlisting divergent and independent interests to attain a valuable objective that everyone believes in, if perhaps for somewhat self-interested reasons.
> "***Cognition***: The process of educating people in preparation for using another technique to attain unity of purpose.
> "***Coercion***: Invoking higher authority to attain a short-term goal. It is most effective in preparation for coalition building OR when a decisive victory is possible."

A coalition is the identification of an objective that everyone believes in. This makes it distinct from the idea of a consensus that possibly nobody believes in. Our dynamic law of cooperation goes beyond the idea that the outcome is just better for everyone. Our proposal makes clear that a *common ground* is required and we have provided a precise definition of what this concept means. This common ground requires knowledge of each ***player's interest flow*** V_j, (a concept analogous to charge density in physics) as well as their payoffs. If there is no alignment, there can be no potential for cooperation. It is not hard to think of examples in city, state or national governments where common ground is found between divergent groups, *Cf.* (Ury, 1993).

An example of this is the treaty that was signed between the United States and the Soviet Union during the Cold War. These countries did not share the same values, social systems or economic systems. They each saw however the distinct possibility that their possession of nuclear weapons could destroy all life on earth. This was their common ground. As a consequence they were able to cooperate on treaties that reduced the stockpiles of weapons without changing their ideologies. They did not cease to be competitors.

A coalition necessarily requires that the *interest flows* of the parties align. The consequence of no alignment is well illustrated when national political parties become so polarized on issues that they become incapable of being the representatives of the people they were elected to serve. From our *decision process theory*, we expect that under no cooperation, the only forces will be those of competition. If there is no code of conduct other than self-interest, the politics become dominated by special interest groups. The system looks more like the interaction of clans, gangs or medieval war lords than a creative cultural, economic and social environment.

Neither our *decision process theory* nor *game theory* envisions that the existence of cooperation turns off the competitive nature of decisions. What does turn off that competitive nature is the establishment of a code of conduct. For example with the political example mentioned above, a positive outcome between parties occurs when they find that within their organization their views are not monolithic. A party might find that most of its members are conservative on fiscal matters but not on social issues. A liberal party might find that most of its members are liberal on social issues but not on fiscal issues. The two parties might find common ground by being fiscally conservative and socially liberal. To get to this type of distinction in the modeling we require our proposed *decision process theory*. It is a process framework that reflects how realistic systems behave. It is clear that such a process description needs to be dynamic since the common ground is usually in flux.

The law of cooperation in our *decision process theory* requires a common ground, an overlap between the *players' interest flows* or payoffs or both. Without such common ground there can be no cooperation. Without cooperation and without a code of conduct we conclude that behavior reverts to purely competitive self-interest. Thus to establish cooperation one needs both the establishment of appropriate codes of conduct and common ground. With cooperation, the possibility exists for the creation of a robust free market in the sense envisioned by (Smith, 1776).

Because we envision a theory in which competition and cooperation play equal roles, we argue that there must be convertibility between utility associated with each. We turn to the question of convertibility and the related issue of opportunity cost in the next section.

7.7 *Dynamic law of opportunity*

Though there is a different utility function for each agent, our *decision process theory* identifies the utility of each agent with an energy contribution to the overall system.

7.7.1 Energy is convertible

Energy is convertible and comparable. This provides the mechanism for exchange which is a new idea and can be subjected to test. It is not equivalent to the elementary assumption that all utility functions are the same. It is based on a specific underlying dynamic process based on the *principle of least action*. This principle gives a ***law of opportunity***, Eq. (3.17), which determines the ***opportunity potential*** g_{ab} between active strategies in terms of the total energy and momentum of the system:

$$\frac{1}{\kappa}\left(\bar{R}_{ab} - \tfrac{1}{2} g_{ab}\bar{R}\right) = -T_{ab} - T_{ab}^{competition} - T_{ab}^{cooperation} \tag{7.16}$$

The cooperation energy momentum tensor, Eq. (3.20) is determined by the cooperation potential γ^{jk} and the competition energy momentum tensor, Eq. (3.19) is determined by the payoff matrix. The left-hand side of Eq. (7.16) is the opportunity energy momentum tensor, which is a function of the opportunity potentials g_{ab} and their gradients. In the absence of competition and cooperation, the opportunity potential is determined by the external set of stresses T_{ab}, which reflect the ***global connectivity*** of events.

We thus allow any number of choices for utility and any set of frames in which to define express the strategies. We posit that the energy and momentum of the decision process can always be determined relative to these choices. This provides the basis for converting the effects from one frame of reference to another. The resultant ***frame effects*** ϖ_{abc} on the acceleration Eq. (7.15) are no less real than the economic cooperation and competitive effects. The frame effects are the consequences of our topological connectivity assumptions about utilities. These frame effects determine the *orientation flux field* tensor Eq. (2.27) and through the definitions the contracted curvature tensor $\bar{R}_{ab}$, Eq. (2.36). We think of the frame effects as being generated by the ***narrative*** being told about what is happening. The narrative provides the framework and so hides the *frame rotation* effects in the way in which the story is spun.

7.7.2 Law of opportunity

Convertibility is closely tied to the game theory concept of ***equilibrium***. From a mathematical perspective, equilibrium may suggest only the existence of fixed points of the dynamic equations. However, as we inquire more about what equilibrium means in game theory, we see that we are talking about the concept of events not changing, of events being *stationary*. Events are *stationary* because of some real attribute of the process, which is often called ***inertia***. Decisions are processes that will tend to continue along whatever path they have been set; including staying at rest if that is where they started. This also implies that things in constant motion remain in constant motion along their initial direction. As with Newtonian and post-Newtonian physics, change occurs only when forces are applied. We thus envision that the ***law of opportunity*** provides for the possibility that utility can be converted to motion and *vice versa*. This possibility is a part of our dynamically extended game theory; specifically, the conservation law, Eq. (7.15).

Concepts such as Nash equilibrium predict future behavior based on the assumption that players should behave rationally. If players have exhibited a behavior in the past, then they are likely to continue that behavior in the future. The longer they have exhibited that past behavior, the more likely they will continue to behave that way (Bayes, 1764). We take a different view of this tendency. We say that the observation of equilibrium is an observation of a real and tangible property of decisions and their connectivity that we call inertia or decision mass. If the tendency is very pronounced then the inertia is very large. Future behavior may in fact remain stable because there are no forces sufficiently strong to move the system. The inertia is not a prediction of the future but a statement about the reality of the decision process based on present and past observations.

The law of opportunity also includes our notion of ***player's interest flow***. We associated *interest flow* with the possibility of a player being a buyer or a seller. Buying and selling are social transactions (Mill, On Liberty, 1947):

> "Again, trade is a social act. Whoever undertakes to sell any description of goods to the public, does what affects the interests of other persons, and of society in general; and thus his conduct, in principle, comes within the jurisdiction of

> society: accordingly, it was once held to be the duty of government, in all cases which were considered of importance, to fix prices, and regulate the processes of manufacture. But it is now recognized, though not until after a long struggle, that both the cheapness and the good quality of commodities are most effectually provided for by leaving the producers and sellers perfectly free, under the sole check of equal freedom to the buyers for supplying themselves elsewhere. This is the so-called doctrine of Free Trade, which rests on grounds different from, though equally solid with, the principle of individual liberty asserted in this Essay."

If we don't sell something but choose to buy, there is an ***opportunity cost*** associated with our choice to not sell in addition to the cost of buying (Mill, 1848). We therefore expect the utilities to be convertible.

From our discussion of the ***law of cooperation*** (section 7.6), we expect that buyer and seller contribute with opposite signs to the (potential) energy, based on a physical and mathematical argument that their gradients produce the forces that generate motion. For example, when a company sets up to sell a product, they become both producers and consumers. Often they have to borrow money to create their business. In our *decision process theory* they create inertia associated with producing and inertia for consuming. We imagine that these distributions occupy different points in strategic space, as illustrated in our prisoner's dilemma example, Figure 5-17. If we think of ***interest flow*** as analogous to charge, since opposite charges would normally attract and come together, we think the same occurs for opposite *interest flow*. Opposite charges would contribute with different signs to the potential energy, so we expect the same for opposite *interest flows*. From a business perspective it makes sense that the tension that keeps these *interest flows* apart is the ***opportunity*** for making a profit. Our *decision process theory* thus provides the insight that production and consumption contribute oppositely to the potential energy of the system. We extend the concept of convertibility to cover *player's interest flow* then as well as inertia.

7.7.3 Opportunity costs

As an example of opportunity cost, we consider the choice of working at a low paying job (production) versus taking a year off to be trained (consuming) to do a higher paying job. If we take the year off we lose the wages of the lower paying job for one year. That is the opportunity cost of taking the training course. At the end of the year however we hope we will be able to find a higher paying job. We see that the opportunity cost assumes something about risk and the future. We certainly take a risk in not working for a year, especially if we already have the lower paying job. The risk is that we may not get hired for the higher paying job at the end of the year. The reward however is that if we do get hired, over the long term we are likely to receive more total income than we would have received had we stayed in our current job.

There are other types of opportunity costs not associated with buying and selling. As an example of such an opportunity, suppose we have a car with known mechanical problems that will make the car inoperative. Do we keep the car or do we junk it? We have researched the cost of the repair of the mechanical problems and have determined the cost to be $2K. If the car breaks down tomorrow, we will be out the $2K as well as having to get the car towed to the junk yard. If we knew for sure the car would break down tomorrow, we would be ahead by simply driving it to the junk yard today. However, we don't know for sure the car will break down. In fact we have been driving the car for the past year with this known problem. If the car drives for another year we save the price of buying a new car. Is the $2K a real cost? What we can say for sure is that it reflects the opportunity tradeoff of getting rid of the car versus keeping the car.

In our *decision process theory*, we analyze the problem as follows. We have data from the mechanic who has service our car and many similar cars with the same problems. Based on what he has told us we have a current assessment of the utility of the car and whether the car will make it to the future. This is not the same as knowledge of that future. We make no assumption that the mechanic is in fact accurate in his assessment. Based on our current assessment of the car's utility we make our decision. If we could iterate the experience, then we would change our assessment of future utility as we gained or lost trust in our mechanic. Our mechanic too might improve. What we know for sure however is the future assessment based on our past experience.

We propose that there is an opportunity utility that expresses the opinion of future utility for any given strategy, one that is neither cooperative nor competitive. So, just as payoffs express the view of how players will respond in a competitive situation, opportunity utility expresses the view of how the future might unfold. This view is not a prediction of the future however; it is simply the current view of that future, just as the payoff matrix is the player's view or utility measure of how all the other players will behave. Since we take utility to be energy, we consider keeping the car or junking the car to reflect (potential) energy. If the potential energy is higher to keep the car we hold on to it. If the potential is lower, we continue to drive it. The potential reflects a reality about our experiences to date with respect to the car. These experiences are about the past and present. To the extent that the process captures the essence of the ongoing decision processes, we do in the end gain knowledge of the future behaviors, though not as probability predictions.

7.7.4 Opportunity utility

We identify the ***opportunity utility*** with the stress tensor in our *decision process theory*, which provides the source term q_a in Eq. (7.15). The opportunity utility results from a tension between alternatives and so depends on the direction in space and time. We use the word tension because we believe it represents the force that moves the decision makers toward or away from choices based on their current knowledge of past behaviors. This is the tension that moves the system towards the future. Opportunity results from the ***elasticity*** or ***connectivity*** between strategic possibilities, such as the production and consumption required of a company to produce a product and make a profit. Decision processes have complex opportunity connections. The challenge of models is to capture the essence of such connections to accurately portray observed effects. We provided a *decision process theory* for the stress tensor in section 7.11.

In many practical cases, we can choose an ideal elastic situation in which the stress tensor is isotropic and characterized by two parameters, an energy density μ and pressure p. In rough terms, the energy density measures the inertia of the system and the pressure is a measure of the opportunity. The force is proportional to the gradient (rate of change) of the pressure with respect to position, $\partial_a p$. More precisely, we get the source term q_a in Eq. (7.15) using Eq. (2.45) in terms of the inertia and opportunity:

$$q^a = h^{ab} \frac{\partial_b p}{\mu + p} \tag{7.17}$$

We characterized the ratio of energy density to pressure as the ***resilience*** α of the system in the prisoner's dilemma, chapter 5, which we argue is a measure of the system's elasticity. If one looks into the dynamic mechanism of the theory, the collocation of what we can call the ***decision mass*** is the manifestation of inertia, *Cf.* Figure 5-3. The pressure gradient is the manifestation of ***opportunity*** and opposes any forces, such as the *player interest flow*, that try to concentrate the decision mass.

As an example of this more precise definition of opportunity, consider a property of projects that as their size grows, there is progressively more overhead. Overhead includes the increased number of managers required to oversee the project. The assumption is that these managers perform no work. They make no widgets or write no code. Because of their addition, the number of people required grows faster than linear with the size of the project. This overhead is a manifestation of resilience (Thomas G. H., 2006); the larger the resilience of the system α, the larger the overhead. Ordinarily one thinks of overhead as a waste of resources. We suggest however that the identification of α is also the measure of the opportunity. An organization that has zero overhead may be incapable of responding to new situations. An organization with high overhead may have trained more people, since many may have been idle and managers may have been skillful in training. When more work shows up unexpectedly, such an organization may be better able to handle the fluctuations. In this regard, it is an interesting fact that organizations oscillate between high and low overhead. Perhaps this is the dynamics between having too much opportunity versus not enough.

7.7.5 The three laws—competition, cooperation and opportunity

We have three strong laws that govern decision making based on competition, cooperation and opportunity. In each case there is an associated potential that is determined based on a presumed connectivity. The competition potential $A^j{}_a$ is determined by the mixed stress $T^j{}_a$, Eq. (3.16):

$$\tfrac{1}{2} g^{bd}\partial_b\partial_d A^j_a + \tfrac{1}{2}\partial_a g^{bd}\partial_b A^j_d + \tfrac{1}{2}\partial_b g_{ae} g^{ec} g^{bd} F^j{}_{cd} - \tfrac{1}{2} g^{bd}\gamma^{jl}\partial_b\gamma_{lk} F^k{}_{ad} = -\kappa T^j_a \qquad (7.18)$$

This is equivalent to Eq. (7.12). The cooperation potential γ^{jk} is determined by the stress components $T^j{}_k$ from Eq. (7.14). The opportunity potentials g_{ab} are determined from the active stress components T_{ab}, Eq. (7.16). The stress tensor thus provides the critical ingredient to the causal relationship between present and future events. It replaces the Bayesian notion of probability for predicting the future. It reflects the existence of ***global connections*** between events.

We find support for global connectivity from Systems Dynamics (Forrester, 1961) and their causal and global perspective. In many practical examples they have been able to identify the connections and find that in fact they are easy to spot. What matters is the mental set to look for them. We turn to this in the next section.

7.8 *Global connections*

We have noted that using physics and differential equations for modeling economic behaviors is quite old (Edgeworth, 1881). In more recent times, the concept of using differential equations to model social phenomena was proposed by (Forrester, 1961) and used by the ***Club of Rome*** (Meadows, Meadows, Randers, & Behrens, 1972, p. 26) to describe what might happen on a worldwide basis given initial conditions on such things as population, birth rates and resource consumption.

> "We too, have used a model. Ours is a formal, written model of the world[22]. It constitutes a preliminary attempt to improve our mental models of long term, global problems by combining the large amount of information that is already in human minds and in the written records with the new information processing tools that mankind's increasing knowledge has produced—the scientific method, systems analysis and the modern computer."

The important insight is that differential equations provide the large scale structure of time and space (Meadows, Meadows, Randers, & Behrens, 1972, p. 24)

> "Although the perspectives of the world's people vary in space and in time, every human concern falls somewhere in the space-time graph. The majority of the world's people are concerned with the matters that affect only family or friends over a short period of time. Others look further in time or over a larger area—a city or a nation. Only a very few people have a global perspective that extends into the future."

The *Club of Rome's* insistence on considering both large distances and large times matches the view of (Hawking & Ellis, 1973), who brought insight into the large scale structure of space and time at the cosmological level. The latter view has provided both philosophical and mathematical guidance to us for creating a *decision process theory*.

We thus see *game theory* as an accurate, but local and provisional view of how strategic decisions are made over short intervals of time and short distances in space. Using the above language of the *Club of Rome*, *game theory* translates well for family and friends, for cities and for states but not all of these simultaneously. Consider the following military example. Traditional armies can envision battles with large armies and armaments. Armies study how the last war was fought. They have trouble envisioning new wars that may be fought under different conditions as exemplified by the urban war currently going on in Iraq. The standard model of behavior as used in *game theory* is not sufficient to help one think out of the box; one needs courageous generals. Our view of a theory that addresses the large scale structures of decisions in space and time is that we must change the standard model to encompass such game-changing strategies, without necessarily relying on the existence of such generals.

Indeed it was our search for a theory that would locally match *game theory* yet allow for large scale structures that led us to theories with new topologies. We needed theories that in some sense were locally

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[22] "The prototype model on which we have based our work was designed by Professor Jay Forrester of the Massachusetts Institute of Technology. A description of that model has been published in his book, *World Dynamics* (Cambridge, Mass.: Wright-Allen Press, 1971)."

flat such as is the shape of the earth, yet would correctly see the global curvature characteristics. The formal mathematical structures with this property are known in mathematical literature (Steenrod, 1951). They have been independently created as bases for physical gauge theories starting with Maxwell and Einstein and used to understand the large scale structures of space and time (Hawking & Ellis, 1973). Such theories combine the power of topology, algebra and differential geometry. In other words our goal was not to replace what has been learned about economic behaviors from *game theory* but to extend those theories.

Before addressing the large scale structures that are possible in such theories and the motivation for expecting such behaviors we obtain from systems dynamics, we note that *game theory* has not entirely relied on the algebraic approach. There have been studies of continuous and learning behaviors for repetitive games (Clemhout & Wan, 1989). There have been approaches that use the physics metaphor to focus on the stochastic nature of economic processes (Murphy, 1965). ***Decision Analysis*** also focuses on the stochastic nature (Howard, Dynamic Programming and Markov Processes, 1964), according to the Wikipedia article on this subject (Howard, http://en.wikipedia.org/wiki/Ronald_A._Howard, 2010). We expressed reservations towards this approach in section 7.3. Though some of these approaches are differential in character, we feel they don't address the ***global connections*** suggested by the *Club of Rome*.

In this context, we feel that our ***decision process theory*** is most closely aligned with the approach suggested by (Forrester, 1961) based on the causal and continuous nature of time. There are many insights to be gained. We start with a summary of the key elements, not the least of which is the availability of improved computational techniques. An important element of great practical advantage is that there are multiple software packages that make application of systems dynamics no more difficult than applying a spreadsheet model to an accounting problem. In this regard, we have found the iThink® software by High Performance Systems (Richmond, 2001) to be particularly illustrative.

7.8.1 System modeling

The general theoretical concept of systems models is that forecasting requires more than correlations, it requires causation and ***closed loop thinking***. Causal thinking is typical of physical models as well. As an example, though early models of weather forecasting relied more on statistical models than on causal flow, current weather predictions are based entirely on a causal model of fluid flow (Friedman, 1989). It might be argued that the maturity of the field dictates one approach rather than another. So although decision making provides an excellent example of a causal forecasting problem, the field is not yet sufficiently mature for these *operational-thinking* techniques. We argue on the contrary that the system thinking is useful even at the earliest stages of thinking about any forecasting problem.

The basic concepts that underlie systems dynamics and systems thinking are ***flows, stocks*** and ***closed loops***. These concepts provide an abstraction of the underlying differential equations that are sufficiently simple to be understood by school children and CEO's (Senge, 1990). In the software, *stocks* are represented as *rectangles*, *flows* as *faucets* and the causal nature or cause and effect is indicated by curves with arrows indicating the flow of time. We give an example Figure 7-3. We model predators, such as foxes and prey, such as rabbits. The number of predators is collected in the stock or reservoir labeled predator, and the number of prey is collected in the stock labeled prey. The number of predators increases in proportion to the number of prey, since they are its food source. Correspondingly, the number of prey decreases in proportion to the number of predators. In each case, the numbers of predators and prey increase due to their own birth rate as set by parameters mu (μ) and lambda (λ), respectively.

The prey, such as rabbits might also increase because of their food source (carrots). The bi-directional character of growth is represented by flows that have arrows that point in both directions. We represent the continuous nature of time and approximate the discrete nature of creation or demise of predators and prey by a continuous amount in their reservoir. The picture we create in terms of flow results in a quantitative description of the populations Figure 7-4, which in this case has unusual structure in no small part due to the additional food source of carrots.

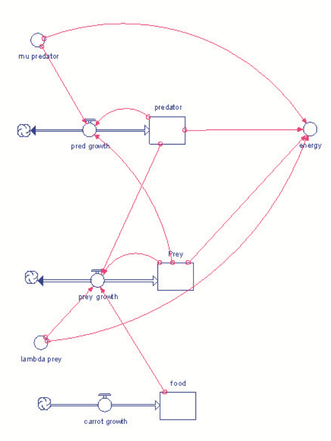

Figure 7-3: Systems Dynamic Predator-Prey

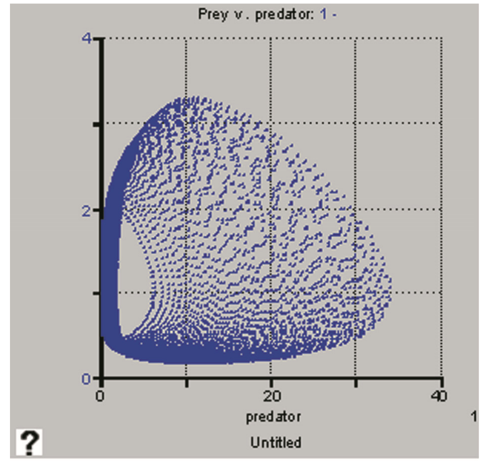

Figure 7-4: Predator Prey populations

What is truly significant about the systems dynamics approach is not its application to the simple problem above, whose simplicity was there only to provide a convenient way to explain the concepts. It forces one to think beyond the initial problem, to all effects that might be significant. For example, in addition to considering foxes and rabbits, we forced ourselves to consider carrots. Without food, the rabbits will die and so then will the foxes. These examples of ***global connectivity*** and ***causal connectivity*** are the key aspects of the *Club of Rome's* study of the world's problems. Events big and small are connected in both time *and* space.

The stocks and flows represent both these causal and global connections. They can be applied to a variety of concepts from physical aspects such as the number of predators and prey, the carrot crop as well as to more psychological or social aspects. For example, in applying this approach to a business, one can model not just productivity and staffing but the burnout rate and thereby learn that burnout can play an important role in understanding delivery times of products. The *Club of Rome* dealt with many issues that have a social character, such as the harm done to the environment by the burning of fossil fuels. Certainly the amount of carbon dioxide is a physical attribute; what kind of attribute is harm?

We thus see clear advantages to the systems approach over statistical or stochastic approaches. To quote from (Richmond, 2001, p. 19):

> …In short, Operational Thinking is a big deal! It's a big deal because, like Closed-Loop Thinking, it has to do with how you structure the relationships between the elements you include in your mental models. Specifically, Operational Thinking says that neither "correlation," nor "impact," nor "influence" is good enough for describing how things are related. Only *causation* will do!"

This operational approach has been applied successfully to real-world situations and the approach produces actionable insight. Real examples involve not a few equations as in the predator-prey example, but hundreds or even thousands of coupled differential equations. Without an expert consultant, it is easy to get lost in the details. We want a systems dynamics approach with its hundreds of equations that also fosters a conceptual understanding of the problem.

7.8.2 Physically based models

The motivation for moving to a physically based model, a model based on a least action principle, is to get such a conceptual understanding; a better handle on the assumptions for the causal and global connections. We want to use the scientific method to improve on the assumptions. Despite conceptual insight, we nevertheless expect that the models will still have lots of equations. Our example of the Prisoner's Dilemma, chapter 5, has hundreds of equations. However, we believe that the concepts are more organized and easier to evaluate, change and/or extend. The example of making accurate weather predictions is an excellent example of both attributes: the fluid flow model involves a large number of equations yet the concepts are orderly and the causal and global connections clear.

In our *decision process theory*, we have emphasized both the causal and global connections between what is known and what decisions are taken. We have created the closed loops and they have the same significant effect in our *decision process theory* as they would be expected to have from an Operations Systems viewpoint. For example, in our *decision process theory*, the flow of decisions creates the values of the payoff fields. The values of the payoff fields dictate the behavior of the flows.

Understanding the consequence of systems dynamics models and physically based models requires significantly more sophisticated computing tools than the original game theory models. The original *game theory* models could be solved as *linear programming* problems: coupled linear (algebraic) equations with constraints. This may have been challenging in 1950 for corporations such as Rand, but is not as challenging today. We have already noted that Systems Dynamics models require stock-flow software (Richmond, 2001). The number and complexity of such equations is challenging for hand-calculations but has been addressed effectively by commercial software. Our solutions to the Prisoner's Dilemma relied heavily on solving the coupled ordinary differential equations using the symbolic programming Mathematica® software by (Wolfram, 1992). To make further progress on the approach outlined in this book, we apply more powerful techniques when there are multiple active strategies and solve the corresponding coupled partial differential equations (Courant & Hilbert, 1962). Progress in computing power makes this extension possible. We have used Mathematica©, with the ***numerical method of lines*** technique for solving partial differential equations in chapters 8 and 11.

We have identified competition, cooperation, inertia, opportunity and global connections as key mechanisms that have their origin in historical discussions and have a place in our *decision process theory*. The next section summarizes how we apply this information to the numerical solution of decision processes.

7.9 *The known behaviors*

Any student who hopes to apply our *decision process theory* must be able to take any economic or decision problem, put it into normal form, use natural units and extract the payoffs, *game values* and Nash equilibriums. The next step is to use this information as the known boundary conditions so that the equations developed in chapter 4 can be applied and solved. We recall the main ideas from section 4.5, which provides a subset of solutions to the field equations:

- Games are specified in the *normal-form coordinate basis* by providing the mixed strategy articulated as flows $E^a{}_o$ for each (active) pure strategy a.
- For each pair of pure strategies there is an outcome or payoff to player j, which we label as $F^j{}_{ab}$.
- We identify the ***players*** as corresponding to the inactive strategies.
- We identify ***true players*** as those that are accountable for some subset of active strategies and ***observer players*** or ***observers*** as those players who are accountable for no active strategy.
- We identify the ***code of conduct*** by stating which of the available active strategies are restricted to be inactive.
- The payoffs and flows are considered as ***known behaviors*** at some point in time and some initial choice of strategies.
- At this point, we may have determined only a subset of the known behaviors. Further research or assumptions may be required to complete the set. We deal with that later.
- Information in the *normal-form coordinate basis* is transformed to the co-moving coordinate basis by means of the *stationary* coordinate transformations, $\left\{E^j{}_o \quad E^j{}_\alpha \quad E^j{}_\upsilon = 0\right\}$, of the

inactive strategies and the dynamic coordinate transformations, $\left\{E^a{}_o \quad E^a{}_\alpha = e_\alpha E^a{}_o \quad E^a{}_\upsilon\right\}$, of the active strategies.

- These transformations obey the equations specified in Table 4-1. As stated above, we identify the active flow transformation $E^a{}_o$ with the *known behavior* strategy flows.
- The *normal-form coordinate basis* payoff fields $F^j{}_{ab}$ are assumed to be specified as part of the *known behaviors*. These are related to persistent attributes $\left\{f^j{}_{\upsilon o} \quad f^j{}_{\upsilon\upsilon'}\right\}$ from Table 4-1, which in turn are determined from the scalar orientation potentials in the co-moving coordinate basis.
- We specify the payoff fields as follows: we use the concept of preferences to identify the spatial components $F^j{}_{mn}$. These preferences, including internal preferences are assumed to exist following the utility arguments in section 7.4. In terms of the assumed known behavior of the un-normalized flow $\zeta^a = \left\{\zeta^m \quad m_0\right\}$, the time component is set $F^j{}_{0m} = {}^1\!/_{m_0} F^j{}_{mn}\zeta^n$ .by the requirement that the product of the payoff field and flow is zero. The time components are thus the normalized values of the *expected payoffs* assuming the known behavior flow.
- The inertial constant m_0 provides a measure for how quickly the system responds to change. This constant is not determined initially, though it may be determined from sensitivity analysis.
- There are different types of orientation potentials, which can be characterized by their rough correspondence to analogous fields in physical models. One grouping given in Table 4-4 is analogous to electromagnetic fields. The streamline solutions correspond to the case that the *electric field* components $\theta_{\upsilon\alpha}$ are zero. As a consequence, the co-moving payoff fields $\omega_{\alpha\upsilon\upsilon'}$ are *stationary*. A second consequence is that these *magnetic field* components are determined by the currents analogous to Ampere's law in physics.
- A second grouping, given in Table 4-5, is related to ***bond forces***. The symmetric *tidal bond* matrix $\omega_{\upsilon\alpha\beta}$ components are *stationary*. These terms are not as well studied in the physics literature and so no direct analogies come to mind.
- The third grouping, given in Table 4-6, provides the inertial stress components. Whereas the orientation potentials describe the rotational and strain aspects, the stresses describe the forces that act on the system.
- To the extent that the stresses are *stationary*, we expect the acceleration components q^υ to be *stationary*. This leads to the charge gradient components $\omega_{\upsilon\alpha}$ being *stationary* as well from Table 4-5.
- It appears at this point that the tidal magnetic components $\omega_{\upsilon\upsilon'}$ might depend on the *proper time*. However, we take the point of view for streamline solutions that these scalar fields as well be *stationary*.

In writing out these steps, we impose no limitation on the number of players or the number of strategies available to each player.

We make no assumption about the codes of conduct other than they must be identified. For each player we assume the existence of at least one code of conduct associated with that player's self-interest. We recall our concept of equivalence that any strategy that is inactive can be considered as an additional player with no associated strategy. Thus the code of conduct is an attribute of every game.

We turn to *game theory* for help in specifying some of the known behaviors, for articulating the intensive form of the game and for an understanding of mixed strategies. Nevertheless our *decision process theory* has distinct differences. The ***known behaviors***, namely the given and appropriate boundary conditions, are used with partial differential equations, to obtain well-behaved solutions.

In chapters 8 and 11 we have obtained numerical solutions for problems with two or more active strategies. We use techniques that have been used in mechanical and electrical engineering on similar problems (Courant & Hilbert, 1962).

As part of determining *known behaviors*, it is helpful to think of *stationary* situations, though not because they are the common place in the real world. Rather we study them because from a study of how *stationary* behaviors change when subjected to an impulse, we learn about the dynamic mechanisms. We provide more details on *stationary* behaviors in the next chapter.

We thus arrive at a practical, calculus-based process for analyzing decisions. As a reminder, our process grew out of an inquiry into *game theory*, which is not only an inquiry about recreational games but about economic behaviors (Von Neumann & Morgenstern, 1944). As such, *game theory* covers much of what should be included in a theory of decisions. We have generalized this approach to a rich dynamic theory of decision processes.

7.10 ***Outcomes***

From this chapter the student should have a broad understanding of classical *game theory* and how that theory is related to our *decision process theory*. Below we provide the detailed outcomes for each section.

From section 7.1, the student will have available a bibliography of relevant discussions from the literature of game theory and economics from which to judge *decision process theory*.

From section 7.2, the student will have learned a theoretical foundation and reasons why a necessary requirement for robust decision processes is the existence of a code of conduct. Without a code of conduct, we can be led into paradoxes such as the prisoner's dilemma and the tragedy of the commons.

From section 7.3, the student will have learned the advantages and pitfalls of applying the concepts of probability and uncertainty to decision processes. The student will have learned that in our *decision process theory*, each player sees an advantage to diversify their portfolio of choices and choose plays according to some frequency distribution of pure strategies, called a ***mixed strategy***. It is not required that this frequency distribution be a prediction of what is likely to happen in the future.

From section 7.4, the student will understand how to identify a different utility function for each agent. The theory identifies the resultant payoff fields with the energy of the system. Energy is convertible and comparable, though utility is not.

From section 7.5, the student will have learned the relationship between the max-min rule of game theory and our dynamic ***law of competition*** for obtaining the flow vectors from the payoffs, Eq. (7.12). The ***economic equivalence principle*** is that our view and the max-min rule are the same for two-person zero-sum non-cooperative games.

From section 7.6, the student will have learned the proposed fundamental ***law of cooperation***, Eq. (7.14). Players contribute differently to this law depending on their ***interest flows***, whether they are ***givers*** or ***takers.*** The law of cooperation in our *decision process theory* requires a ***common ground***, an overlap between the *players' interest flows* or payoffs or both. Without such common ground there is no cooperation. Without cooperation and without a code of conduct we conclude that behavior reverts to purely competitive self-interest. Thus to establish cooperation one needs both the establishment of appropriate codes of conduct and common ground. With cooperation, the possibility exists for the creation of a robust free market in the sense envisioned by (Smith, 1776).

This provides in section 7.7 a mechanism, the ***law of opportunity***, for exchange which is a new idea that extends the concept of value. It is not equivalent to the elementary assumption that all utility functions are the same. It is based on a specific underlying dynamic process based on the *principle of least action*. In a dynamic system, we can convert utility between players, as well as convert between ***interest flow, inertia*** and ***opportunity***.

From section 7.8, the student will have learned the necessary existence of global connections and the elementary aspects of systems dynamics and the importance and generality of both the causal and global connections. The student will have learned the importance of closed loops and they have the same significant effect in our *decision process theory* as they would be expected to have from an Operations Systems viewpoint. For example, in our *decision process theory*, the flow of decisions creates the values of the payoff fields. The values of the payoff fields dictate the behavior of the flows.

In section 7.9, the ***known behaviors***, namely the given and appropriate boundary conditions, are used with partial differential equations, to obtain well-behaved solutions. This section ties the ideas of game theory to the main body of work in the earlier chapters of the book.

7.11 *Exercises*

1. Show that the model calculation in chapter 5 can be changed slightly to conform to a decision process in which each player has two strategies, honor the code of conduct (H) and break the code of conduct (B). The inactive strategies will be $\{B_1 \quad B_2 \quad H_1 + H_2\}$ and the active strategy will be $H_2 - H_1$. Furthermore show that the most general *initial* payoff matrix for each player can be chosen to have the following form, where the order of the rows and columns is $\{H_2 \quad B_2 \quad H_1 \quad B_1 \quad t\}$:

$$F^1{}_{ab} = \left(\begin{array}{cc|cc|c} 0 & y & \frac{1}{2}(-x-y-z) & \frac{1}{2}(-x-y+z) & -\frac{w}{m} \\ -y & 0 & \frac{1}{2}(-x+y-z) & \frac{1}{2}(-x+y+z) & -\frac{w}{m} \\ \hline \frac{1}{2}(x+y+z) & \frac{1}{2}(x-y+z) & 0 & -z & \frac{w}{m} \\ \frac{1}{2}(x+y-z) & \frac{1}{2}(x-y-z) & z & 0 & \frac{w}{m} \\ \hline \frac{w}{m} & \frac{w}{m} & -\frac{w}{m} & -\frac{w}{m} & 0 \end{array}\right)$$

$$F^2{}_{ab} = \left(\begin{array}{cc|cc|c} 0 & -z & \frac{1}{2}(x+y+z) & \frac{1}{2}(x-y+z) & \frac{w}{m} \\ z & 0 & \frac{1}{2}(x+y-z) & \frac{1}{2}(x-y-z) & \frac{w}{m} \\ \hline \frac{1}{2}(-x-y-z) & \frac{1}{2}(-x-y+z) & 0 & y & -\frac{w}{m} \\ \frac{1}{2}(-x+y-z) & \frac{1}{2}(-x+y+z) & -y & 0 & -\frac{w}{m} \\ \hline -\frac{w}{m} & -\frac{w}{m} & \frac{w}{m} & \frac{w}{m} & 0 \end{array}\right) \tag{7.19}$$

2. Write the TIC-TAC-TOE game in extensive form.
3. Rewrite the TIC-TAC-TOE game in intensive form. Show that it is a strictly determined game. What is the Max-Min solution?
4. A pair of blue bombers is on a mission: one carries the bomb and the other carries equipment. The Blue/Red payoff matrix is $\begin{pmatrix} 60 & 100 \\ 100 & 80 \end{pmatrix}$ (Williams, 1966, p. 47). Determine the mixed strategies for each. The rows and columns are labeled: Blue 1=bomb carrier in less-favored position; Blue 2=bomb carrier in favored position; Red 1=attack on less favored position; Red 2=attack favored position.
5. We are told the following river tale (Williams, 1966, p. 50): "Steve is approached by a stranger who suggests they match coins. Steve says that it's too hot for violent exercise. The stranger

says, "let's just lie here and speak the words 'heads' or 'tails'—and to make it interesting I'll give you \$30 when I call 'tails' and you call 'heads', and \$10 when it's the other way around. And just to make it fair—you give me \$20 when we match." The payoff matrix is $\begin{pmatrix} -20 & 30 \\ 10 & -20 \end{pmatrix}$; determine the mixed strategies for Steve and the stranger.

6. The attack-defense game (Williams, 1966, p. 51): "Blue has two installations. He is capable of successfully defending either of them, but not both; and Red is capable of attacking either but not both. Further one of the installations is three times as valuable as the other." The attack or defense of the lesser installation is labeled "one," the other is "two." The payoff matrix for blue is $\begin{pmatrix} 4 & 1 \\ 3 & 4 \end{pmatrix}$. Determine the mixed strategies for each.
7. The music hall problem (Williams, 1966, p. 52): "Sam and Gena agree to meet outside the Music Hall at about 6 o'clock on a winter day. If he arrives early and she is late, he will have to drive around the block, fighting traffic and slush, until she appears. He assigns to this prospect a net worth of -1. If she arrives early and he is late, she will get very cold and wet. He estimates his joy-factor in this case as -3." The payoff matrix is $\begin{pmatrix} 0 & -1 \\ -3 & 0 \end{pmatrix}$. Determine the mixed strategies for each.
8. The huckster (Williams, 1966, p. 56): "Merrill has a concession at the Yankee Stadium for the sale of sunglasses and umbrellas. The business places quite a strain on him, the weather being what it is. He has observed that he can sell about 500 umbrellas when it rains and about 100 when it shines; and in the latter case he also can dispose of 1000 sunglasses. Umbrellas cost him 50 cents and sell for \$1; glasses cost 15 cents and sell for 50 cents. He is willing to invest \$250 in the project. Everything that isn't sold is a total loss (the children play with them). The payoff matrix is buying versus selling with positions rain and shine: $\begin{pmatrix} 250 & -150 \\ -150 & 350 \end{pmatrix}$. Determine the mixed strategies.
9. Write the game paper-scissors-stone game in extensive and intensive form. Show The game consists of two players, each of whom chooses one of the three choices: paper, scissors or stone. The payoffs are that scissors cuts paper, paper covers stone, and stone dulls scissors. This game is not strictly determined. Determine the max-min mixed strategy solution *Cf.* (Williams, 1966, p. 98) who suggests the following payoff matrix for each player, where the rows and columns are labeled scissors, paper and stone: $\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$.
10. Determine the optimum strategy for playing the game of Morra (Williams, 1966, p. 163). There are two players. Each player holds up either one, two or three fingers and simultaneously guesses what the other player will display. If just one person guesses correctly, the payoff is the number of fingers held up. There are 9 pure strategies for each player that can be labeled with two numbers, the number of fingers held up and the guess as to what the other player will do. The pure strategy 13 is to hold up 1 finger and guess 3. Show that the *stationary* behavior involves only three of the pure strategies, 13, 22 and 31. Show that they should be played in the ratio 5:4:3.