WOLFRAM TECHNOLOGY CONFERENCE

Decision Process Theory

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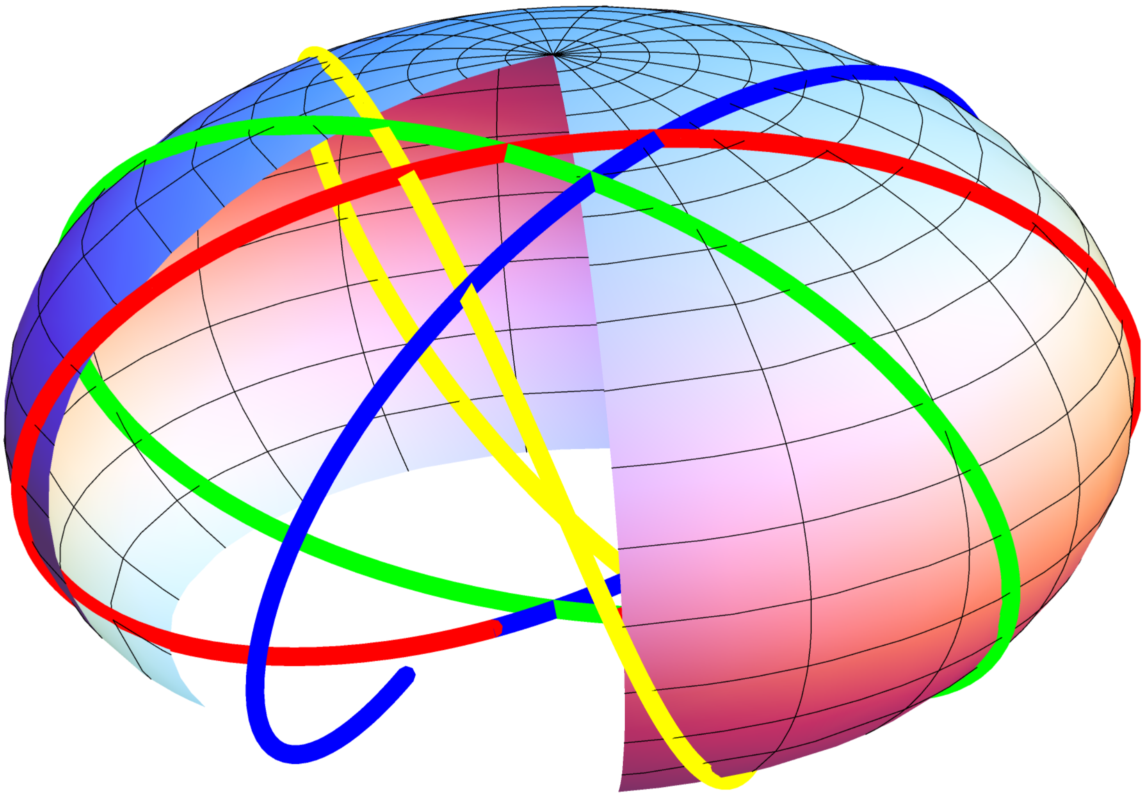
Introduction

I present a quantitative theory of the time dependence of decision processes based on the differential geometry techniques used in such diverse applications as electrical engineering, meteorology and general relativity. Firmly grounded in mathematics, the theory provides insights into such diverse non-mathematical topics as the prisoner's dilemma in game theory, ethical issues of the tragedy of the commons and current day economic behaviors: *Cf.* decisionprocesstheory.com.

Choice and geodesics

A compelling approach to economics and ethics has been the idea that we try to maximize or optimize something: utility, happiness, etc. We look for stable paths, equilibrium conditions (Nash equilibrium). Often it is convenient to treat optimizations by appeal to geometry. We are familiar with geodesics on the earth, but may need to be reminded that for realistic applications, geodesics may be more complicated; even for the earth, it is more complicated since the earth is not round but somewhat flattened. We apply the theory of differential geometry (the study of geodesics) to the theory of making decisions.



Conceptual Basis

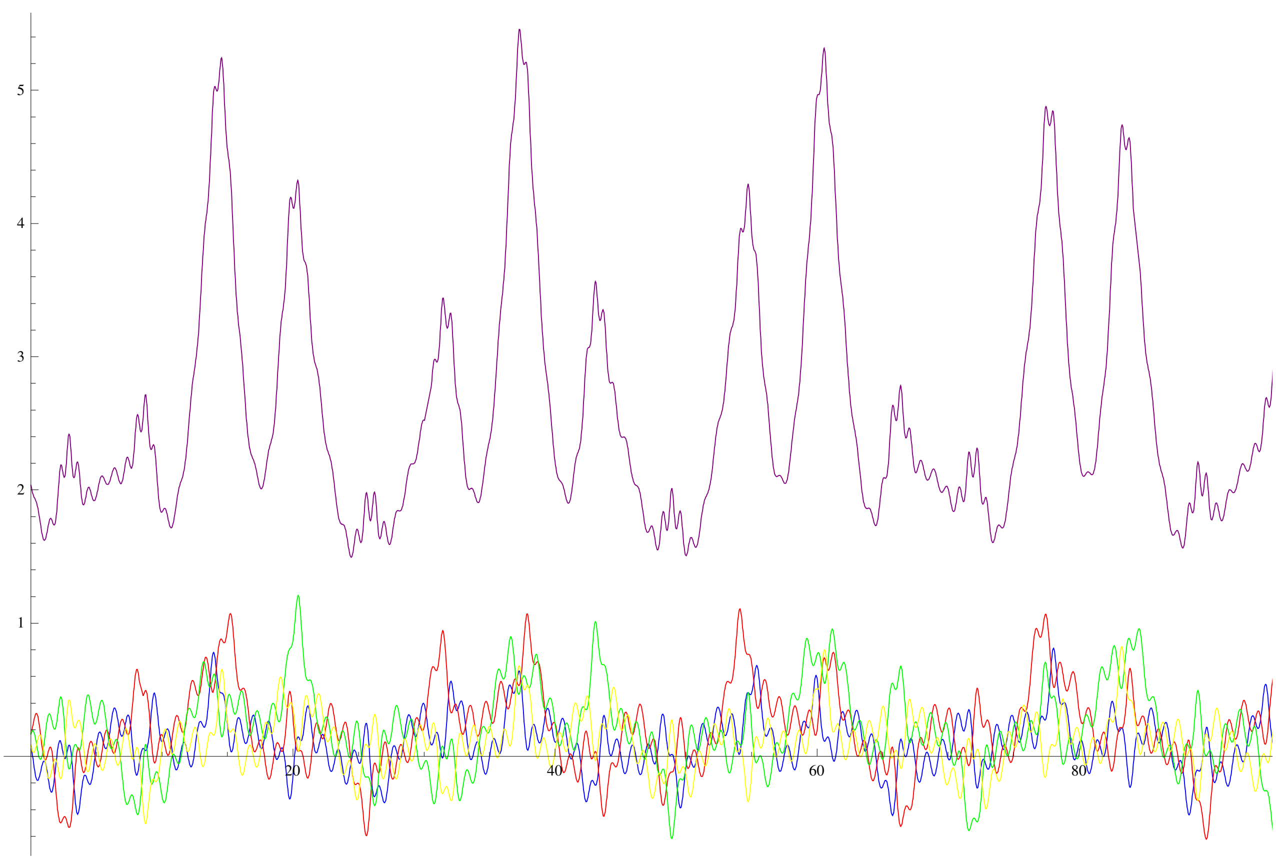
- Decisions are physical processes--non stochastic, non-Bayesian so history matters
- Like the earth, in some sense decisions are locally flat--principle of least action
- Though complicated, certainly an example of Lagrange's principle of least action
- What are the symmetries, what are the active variables, what are the constraints?
- Normal Form pure strategies are the “active” variables
- Need some conceptually simple example(s)—a “hydrogen atom” of decision processes

Choice and harmonics

For our purposes, we consider choice not as static but as dynamic, such as for the stock market. Note the connectivity, the elasticity

Decision Process Theory

Here is what happens in this theory--Many similarities to the stock market picture: again note the connectivity, the elasticity



- The approach starts with the Theory of Games, but departs with significant deviations and help from physical and engineering theories.

- These ideas appeared first in Kauffman's knot series: Thomas, G. H. (2006), Geometry, Language and Strategy, New Jersey, World Scientific.
- Textbook: Thomas, G. H. (2012), The Dynamics of Decision Processes, decisionprocesstheory.com.
- Both with Partnership Program Support from Wolfram Research.

Key distinctions that arise in this theory

Mathematics is necessary for a dynamic theory but not sufficient. Mathematics provides the language of relationships. In addition, we need distinctions that give content. Moreover these distinctions must be tied to the field of study. I have found the following distinctions to be a useful start, taking them from multiple examples and models,with extensive borrowng from game theory, engineering, gauge theories and differential geometry.

Distinctions

1. ***Causality:*** Decision processes are causal not stochastic.
2. ***Aggression and limits to greed***: Decision processes are engaged in by players that may engage more or less strongly as a function of the dynamics. If one player engages more strongly than all the others as indicated by frequency of making decisions we say that player is acting aggressively. In model calculations, we see that aggressive behavior is tied to a greedy value (negative) for the engagement or charge. The other player (in a two person game) exhibits an accommodating (positive) value. We find that there are limits to both greedy and accommodating behaviors.
3. ***Network connectivity:*** Decision processes occur in a societal context, so that decisions are connected to each other not only across time but across strategic distance. This gives rise to a variety of distinctions associated with gradient effects: pressure, network compression, bounce and elasticity. In this concept we find support for the three W's: work, wisdom and wealth. We use these as the basis for successful decision structures.
4. ***Still point and Free fall***: Decision process theory will have solutions in which behavior along a path will have a still point (section 8.4) at which the active geometry acceleration is zero. This generalizes the notion of Nash equilibrium from game theory and allows us to use game theory as a baseline for composite decision behavior. A special case of still point behavior is one in which the strategies appear to be in a flat space; the acceleration is determined entirely by the rate of change of the strategy flow. This free fall behavior gives rise to harmonic behaviors with frequencies set by the characteristic payoff.
5. ***Harmonic standing wave:*** Harmonic standing waves can be used to drive the system and help illuminate the free fall harmonics. A general solution can have an arbitrary superposition of harmonic standing waves. The relative weights of the superposition is fixed by the initial conditions. A system can be studied by driving it with forced standing wave harmonics, which will illuminate the underlying free fall harmonics.
6. ***Acceleration***: We expect to see acceleration in decision processes; we expect the flow of decisions to change in direction or in magnitude. These changes may provide the most direct means to measure the new distinctions in decision process theory.
7. ***Code of conduct:*** Preserved and conserved cooperative behaviors.
8. ***Entitlement***: A player acts alone, empowered without regard to other player behaviors; egoistical behavior. This is the first component that governs a player's choice.
9. ***Engagement:*** The coupling or engagement to the characteristic payoff is a new dynamic distinction. The product of the engagement and the characteristic payoff is the second component that governs a player's choice.
10. ***Player Interest:*** The player interest for each particular strategy changes a player's engagement and generalizes the notion of value from game theory.
11. ***Player Passion***: A player's passion for a particular strategy may overpower objective choice and determine entitlement.
12. ***Mutual player support:*** Decision processes are governed in part by player cooperation driven by mutual player validation forces, which play as important a role in governing behaviors as do competitive valuation forces.
13. ***Hidden-in-plain-sight***: In decision process theory, we expect that some harmonic standing waves may be aspects of cycles that are hidden-in-plain-sight, cycles that needs to be incorporated systematically into the solution. Until now, such effects are unseen; distractions we remove when we attempt to see the big picture.

Elastic Behaviors

Key Elements of Decision Process Theory

• Forget "Bayesian" Farmer's Almanac approach which throws away useful information about the past

• Take a systems dynamics approach, with all relevant economic and psychological aspects

• Take a snapshot of the analogs of the wind, pressure and temperatures over the earth

• What will these values be in an hour? A day? A month?

• Use local behaviors from the elastic mechanics to provide differential properties

• Integrate these properties over time to produce the results

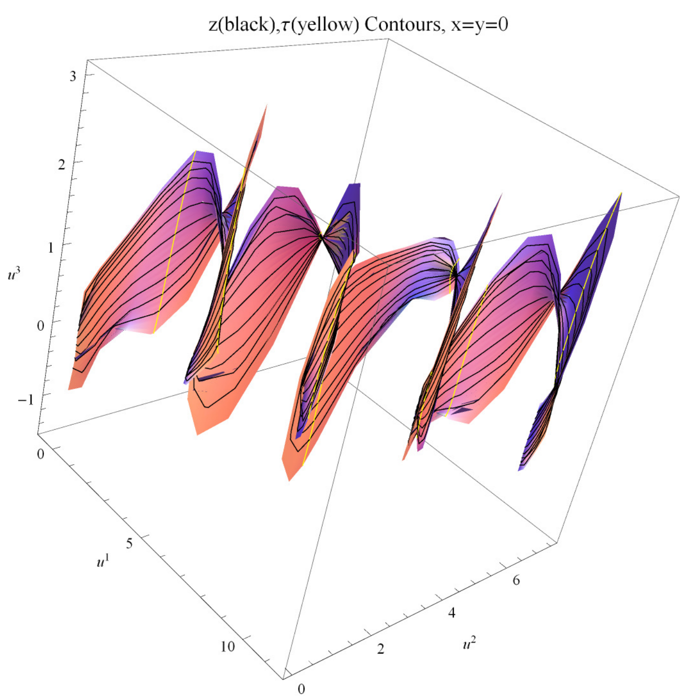
• Look for the analogs of stresses and pressure--these describe the connectivity of the system

What might an application of the theory look like?

Game theory example: Consider two armies, one that attacks (Red) and one that defends (Red). Assume each army has two options, such as to defend two possible installations and to attach the two possible installations.From a game theory perspective there would be a single optimal strategy called the Nash equilibrium. Now assume that such a steady state solution is only one possibility. For the others, consider solutions in which one strategy is a code of conduct (a symmetry) about the overall frequency of play for the sum of all strategies. That leaves three strategies that are active and can change in time.

ATTACK DEFENSE MODEL--Variations in time and space

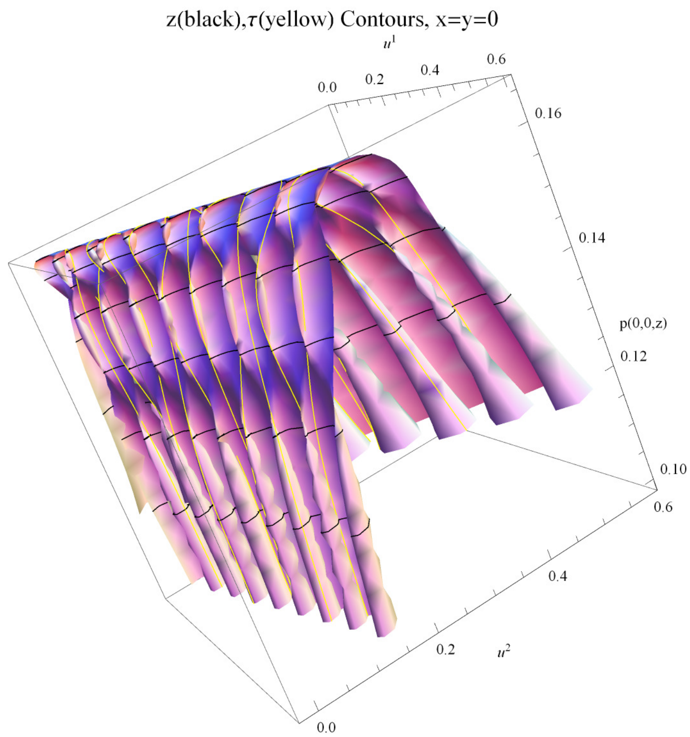
Blue and Red each have a strategy they alone control, u^1 , u^2 and one strategy they control jointly u^3, which is how often they engage relative to the other. The absolute amount of engagement is assumed to be fixed by their code of conduct. Ordinarily these strategies change with time τ, but can be viewed in a frame of reference in which moves with the flow, the co-moving frame; in this case the strategies transform to x, y, and z. Here is the result of the model calculation:



The theory agrees with what many authors have suggested, which is that events in time are connected not only in time, but are connected across space, including the abstract space of strategies (which is nothing more than a mapping from real space). The strength of connection is given by the average stress or pressure.

ATTACK DEFENSE MODEL--Pressure

Here is the pressure. If you examine the figure carefully you will see that the line of constant time represents a standing wave behavior. In theories of gravity this would be a gravity wave. In water and other elastic media, this is a standing wave. Like a pond of water, there will be solutions that are totally still superposed onto ones that become increasingly complex harmonics. What really happens is set by initial conditions.



Prisoner’s Dilemma

Another test of these ideas is to consider a problem whose solution doesn’t come out properly with existing theories. We take the prisoner’s dilemma as such an example. Each prisoner is faced with the same dilemma. They are accused of a crime, separated and questioned. If neither confesses, they each receive a light sentence. If one confesses and the other doesn’t, the one who confesses receives the maximum sentence. If both confess, they each receive a hard sentence but less than the harsh one. The dilemma is that in the real world, prisoner’s often collude and choose to not confess getting the best outcome. In game theory however, each chooses to confess since that corresponds to the Nash equilibrium. These concepts are converted into numerical utilities and payoffs.

Player 1 Payoffs

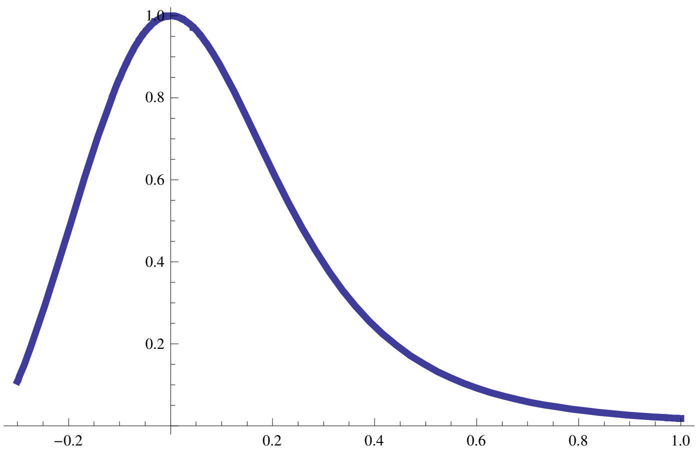
F^1_{ab}	N_2	C_2	N_1	C_1	t
N_2	0	0	$1/10$	0	0
C_2	0	0	1	$9/10$	$-9/(10\zeta^0)$
N_1	$-1/10$	-1	0	0	$1/\zeta^0$
C_1	0	$-9/10$	0	0	$9/(10\zeta^0)$
t	0	$9/(10\zeta^0)$	$-1/\zeta^0$	$-9/(10\zeta^0)$	0

Player 2 Payoffs

F_{ab}^2	N_2	C_2	N_1	C_1	t
N_2	0	0	$-\frac{1}{10}$	-1	$\frac{1}{\zeta^0}$
C_2	0	0	0	$-\frac{9}{10}$	$\frac{9}{10\zeta^0}$
N_1	$\frac{1}{10}$	0	0	0	0
C_1	1	$\frac{9}{10}$	0	0	$-\frac{9}{10\zeta^0}$
t	$-\frac{1}{\zeta^0}$	$-\frac{9}{10\zeta^0}$	0	$\frac{9}{10\zeta^0}$	0

Pressure

In decision process theory, again the pressure is key. One type of solution collects at what would be the equlibrium. However there are many types of solutions including those in which each of the prisoners choose to not confess.



Code of Conduct

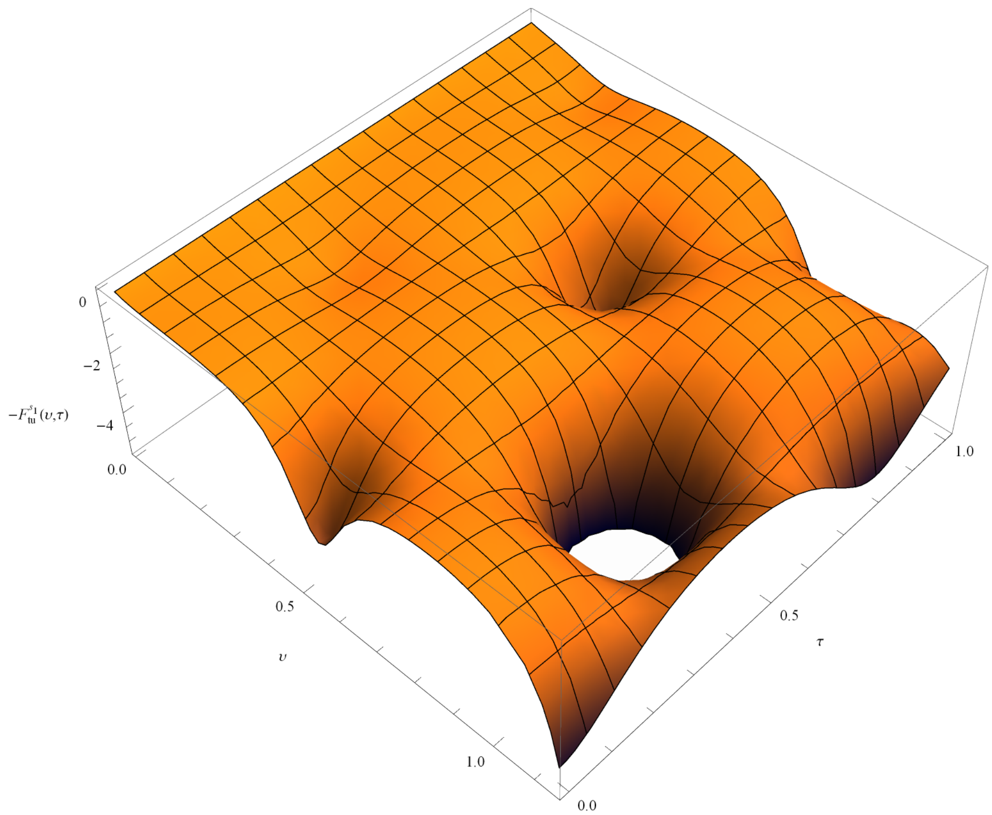
What holds these solutions in place?

- Code of Conduct--agreements between players
- Market (payoff) forces generalizing game theory effects
- Elastic (stress, pressure) network forces
- Acceleration forces--the inertia of going in a certain direction

Game Theory and Metrics

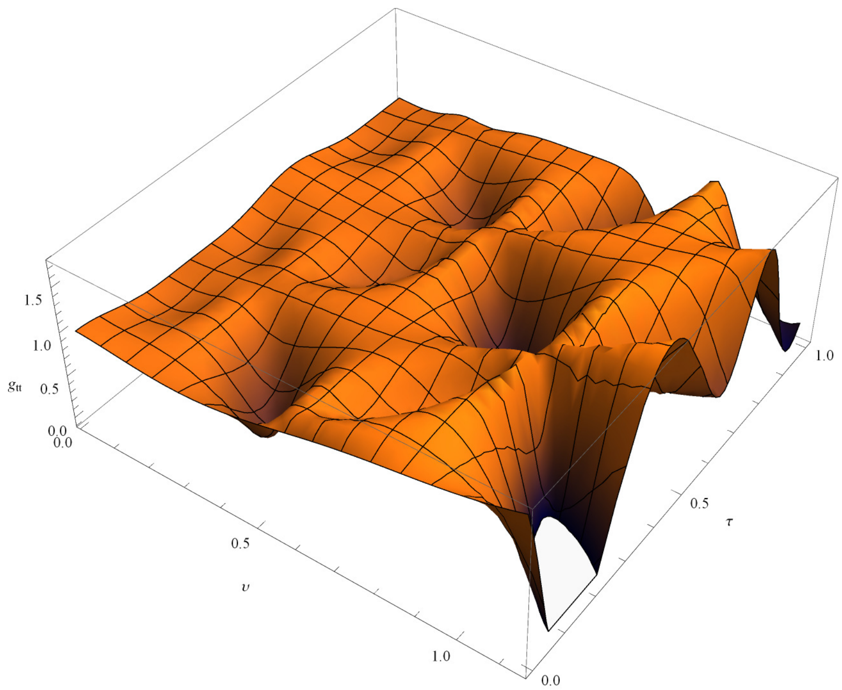
PRISONER’S DILEMMA MODEL--Payoffs

These may vary in time and space!



PRISONER'S DILEMMA MODEL--Metric

Acceleration forces determined by a metric field that generalizes payoff fields. Payoff fields are the measure or utility of a strategy with a player. Metric fields generalize this to the utility between the same or different strategies, strategies and time, or just time. Here is the metric for time. It can be thought of as a gravitational potential in which events get trapped in the valleys.



Conclusions

A dynamic theory can be quantitative and predictive

- Though choices depend on frequency, the change over time is causal and not stochastic
- Not only are linear effects possible but chaotic and catastrophic effects are possible (“storms”)
- The models can be calculated using *Mathematica*--Coupled Elliptic partial differential equations (need more help here)

The theory is not Bayesian

- What happened in the past DOES change the present
- Cause and effect determine future behaviors

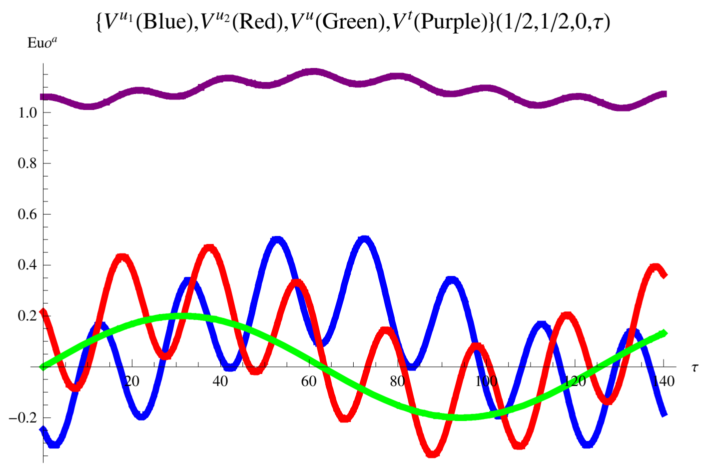
The distinctions involve both time and network connectivity

- What happens in the network now determines what happens in the network later
- Collect too much in one place and it will explode

Potential applications

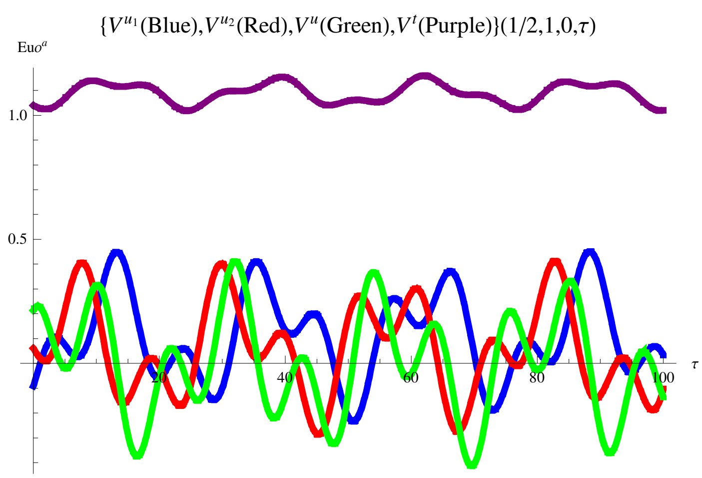
Market cycles

- Applications are approached from a new perspective
- Sustainability--short and long cycles
- As with weather predictions--properties now known everywhere predict properties later, everywhere
- Understand a system by looking at its steady state and harmonic properties--Systems Dynamics approach



Dynamic Games with multiple players and multiple strategies

- Any game theory application--two person two strategy game--examples shown here
- Extend game theory to more than two players with multiple strategies--May stretch *Mathematica* a bit more
- Extends game theory to include new attributes about players/agents



Systems Dynamics applications

- Model systems behaviors--e.g. see software delivery schedules
- As with systems dynamics, psychological or individual concepts play a role--code of conduct in the prisoner's dilemma

Commercial: See my web site for more details

- decisionprocesstheory.com
- See the white papers on my site in general and the The Dynamics of Decison Processes in particular